Learning to erase quantum states

thermodynamic implications of quantum learning theory

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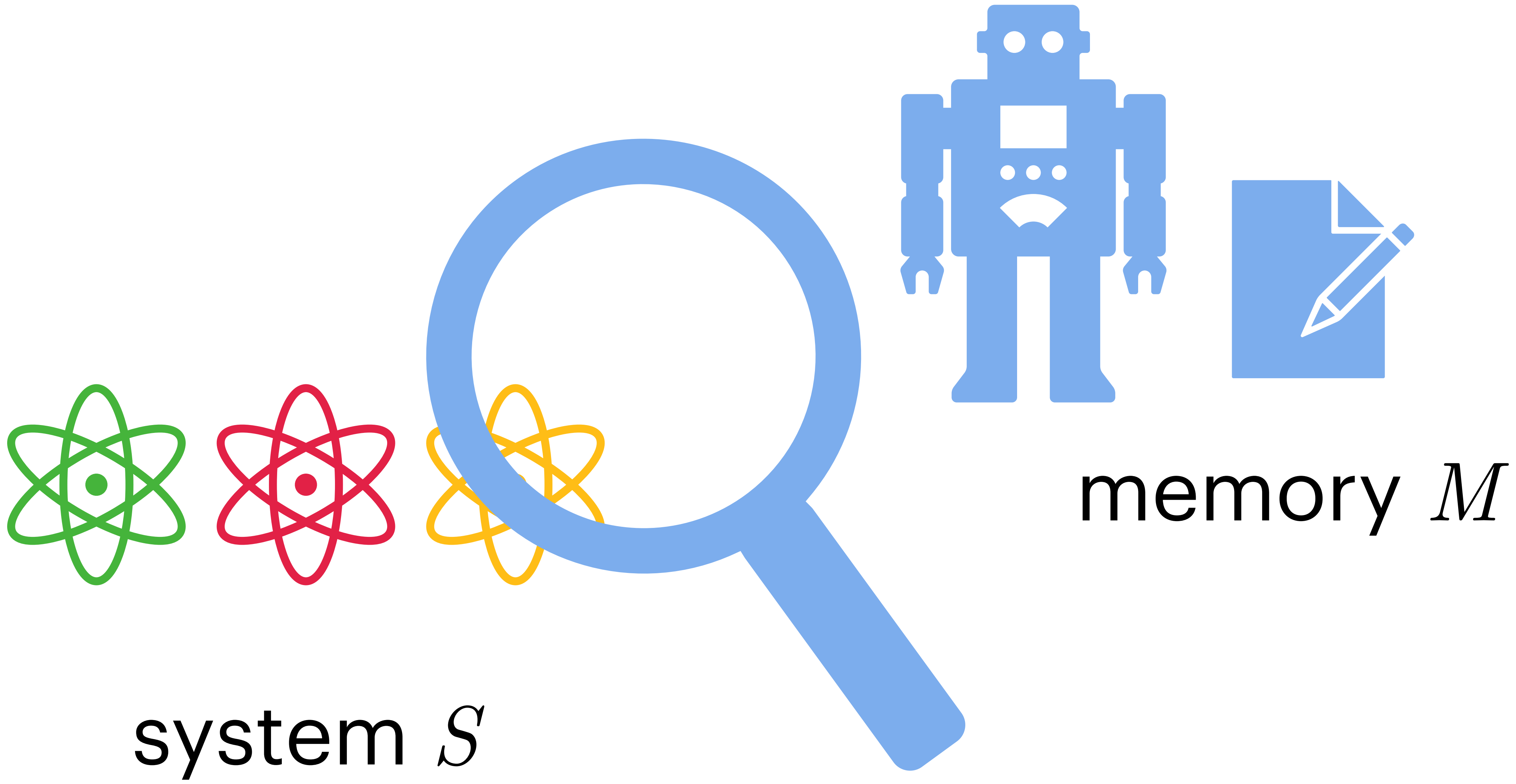
Joint work with Yuzhen Zhang, John Preskill

arXiv:2504.07341

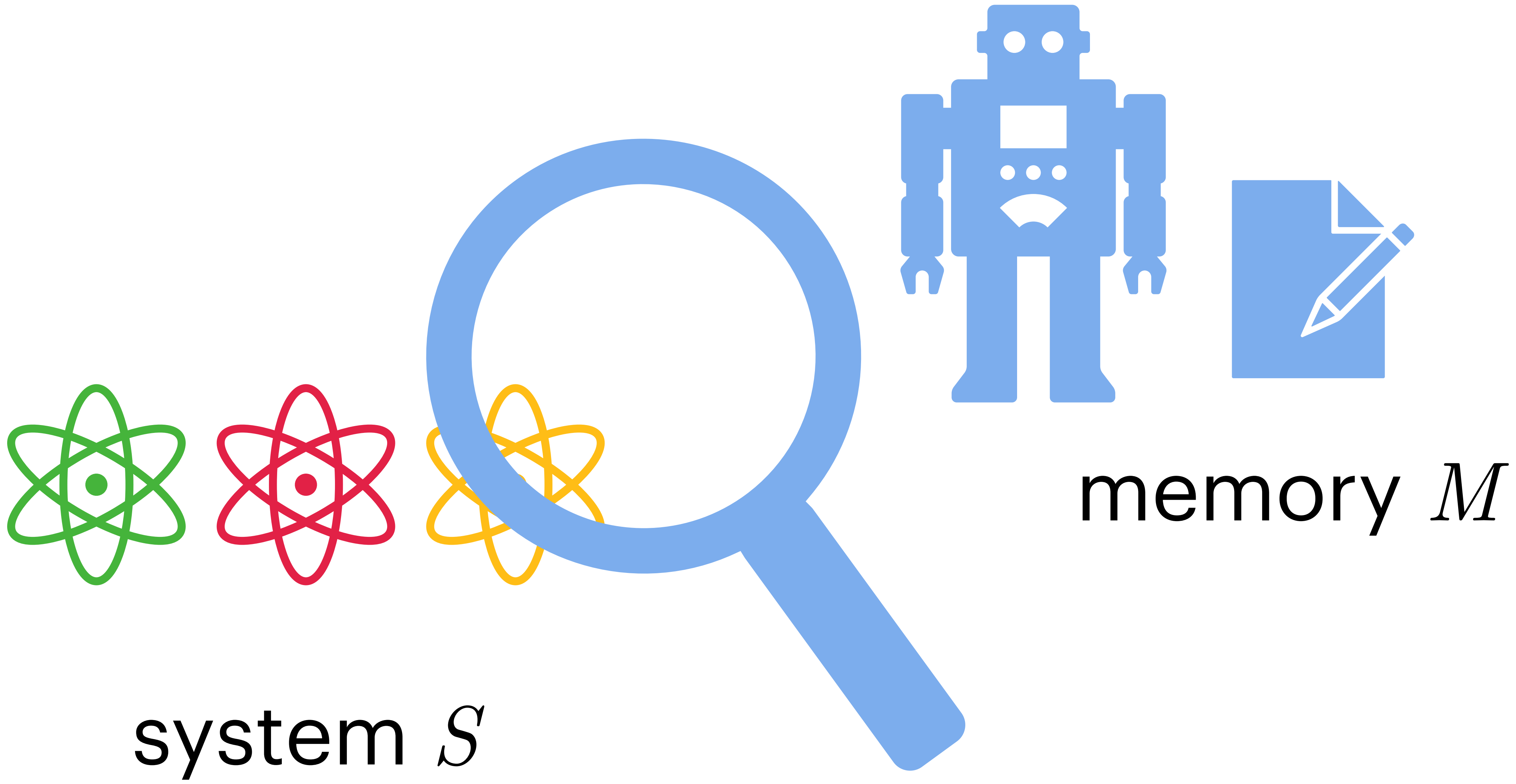
IQIM Caltech UC SANTA BARBARA aws

Motivation



Learning is a physical process.

Motivation



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Questions:

1. *What physical properties does learning itself have?*

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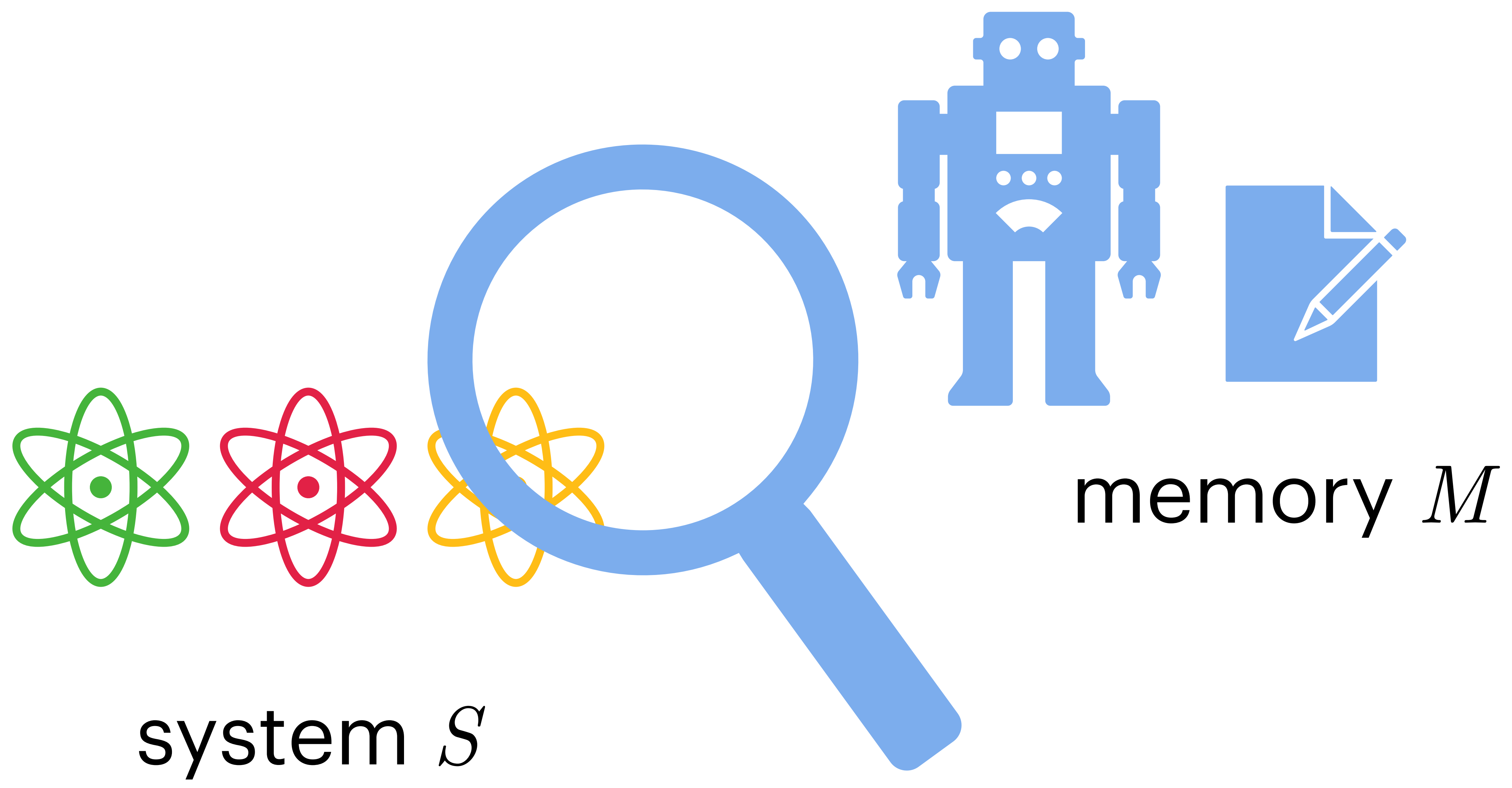

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1. *What physical properties does learning itself have?*

2. *What tangible physical consequences do abstract learning processes have?*

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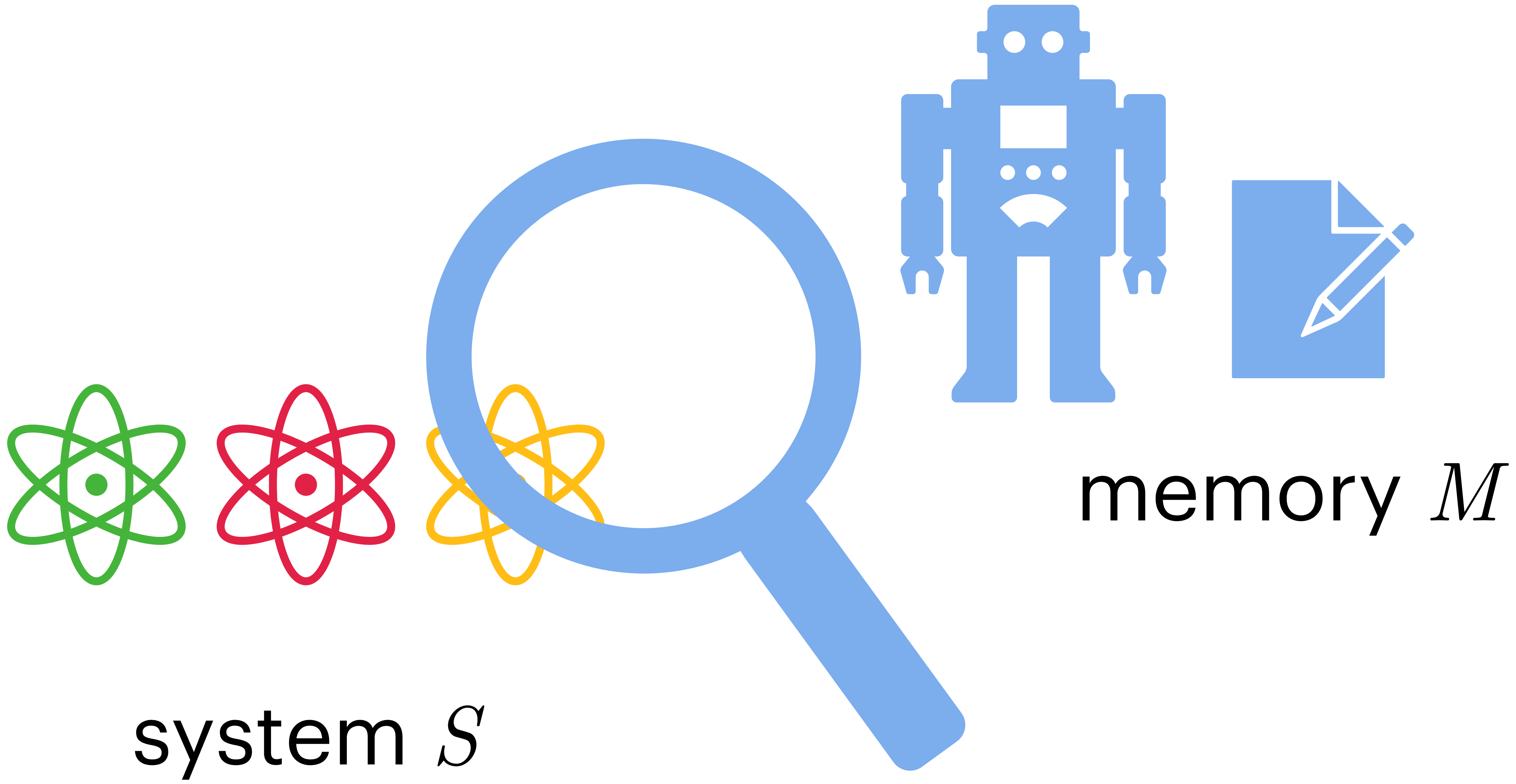

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   Does our (in)ability to learn impact the amount of physical resources needed for certain tasks?

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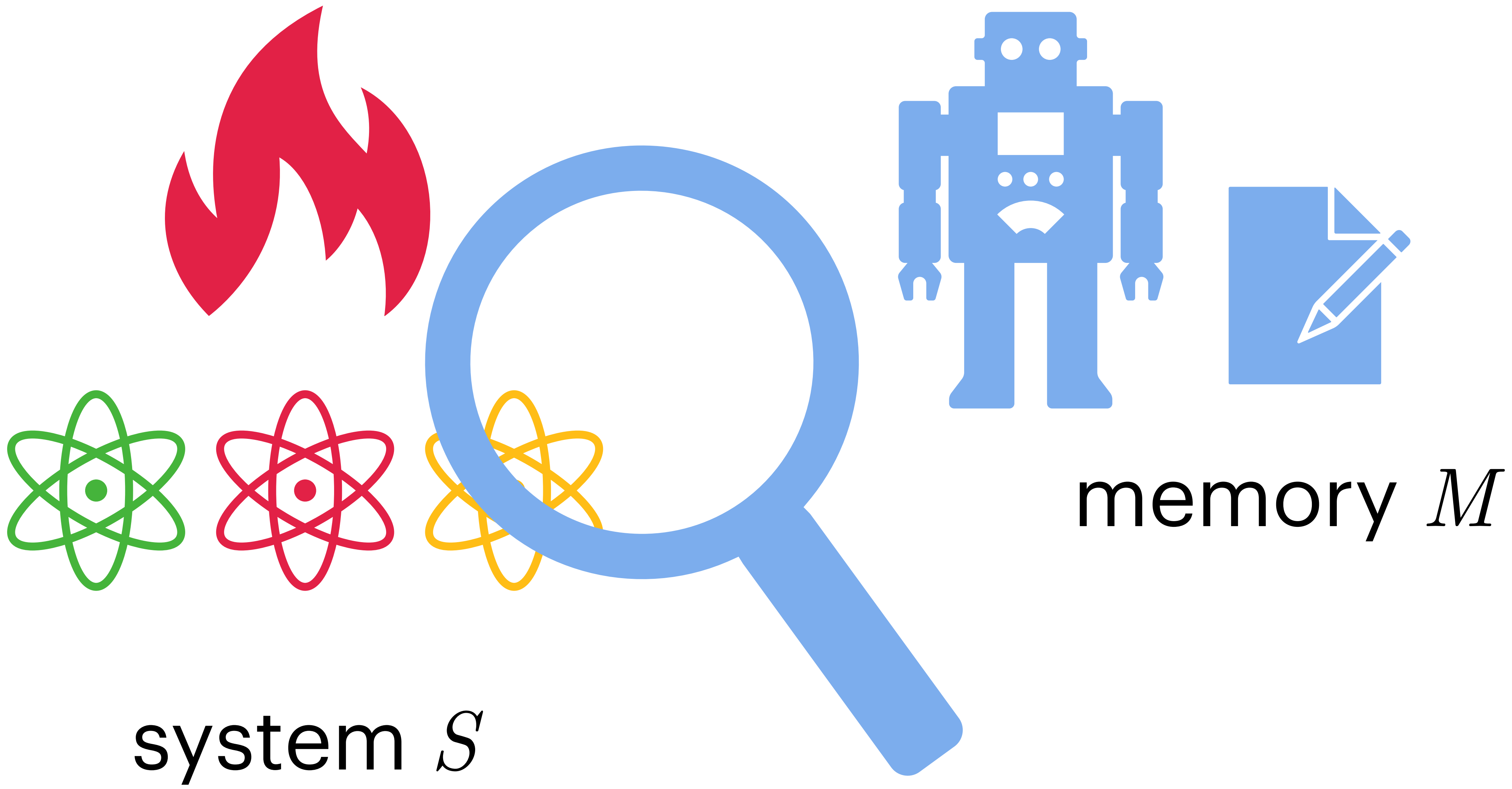


Learning is a thermodynamic process.

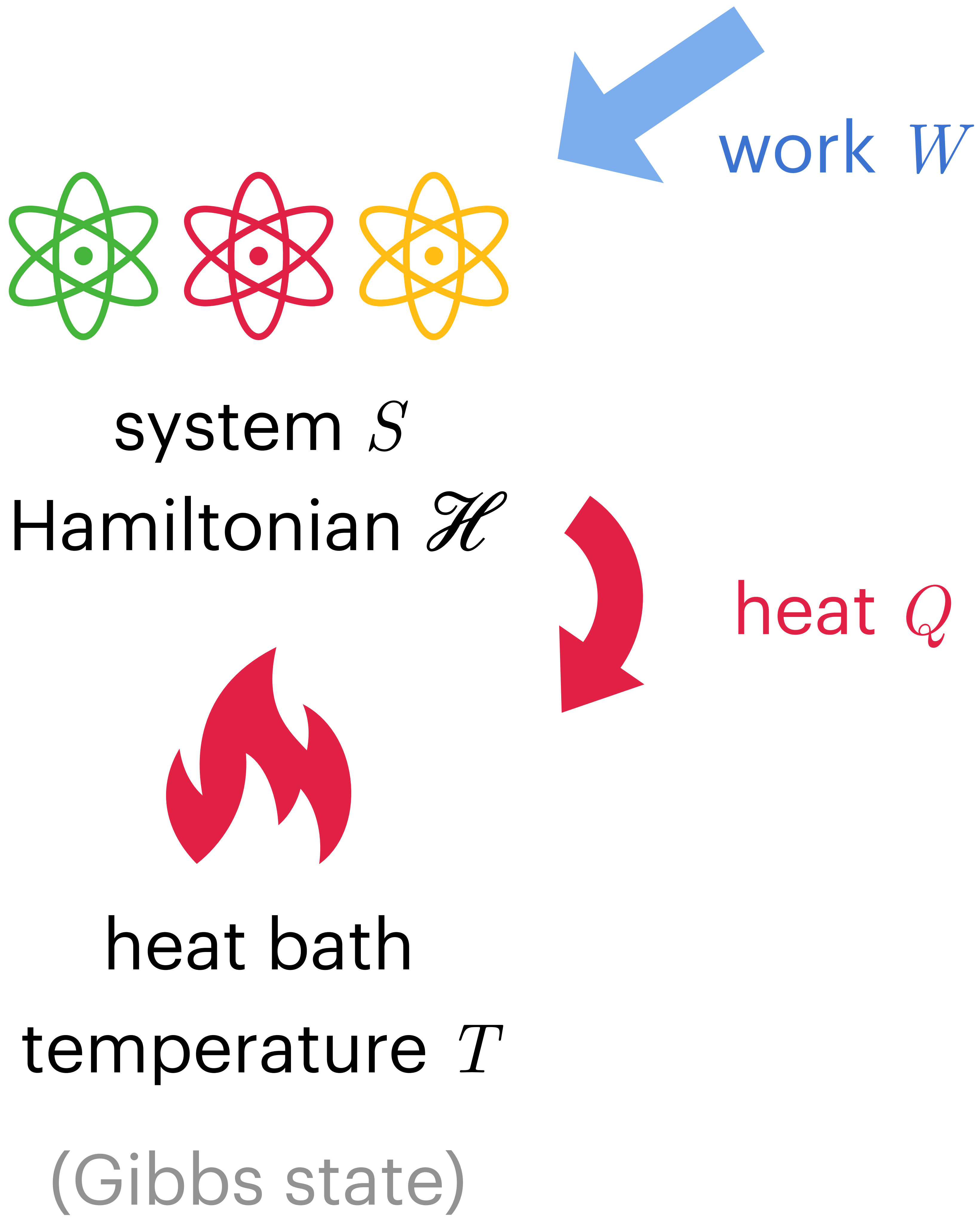
Questions:

1. *What physical properties does learning itself have?*
   No fundamental energy cost.
2. *What tangible physical consequences do abstract learning processes have?*

   Does our (in)ability to learn impact the amount of physical resources needed for certain tasks?

   Yes! Energy cost/gain in erasure/work extraction.

Thermodynamics

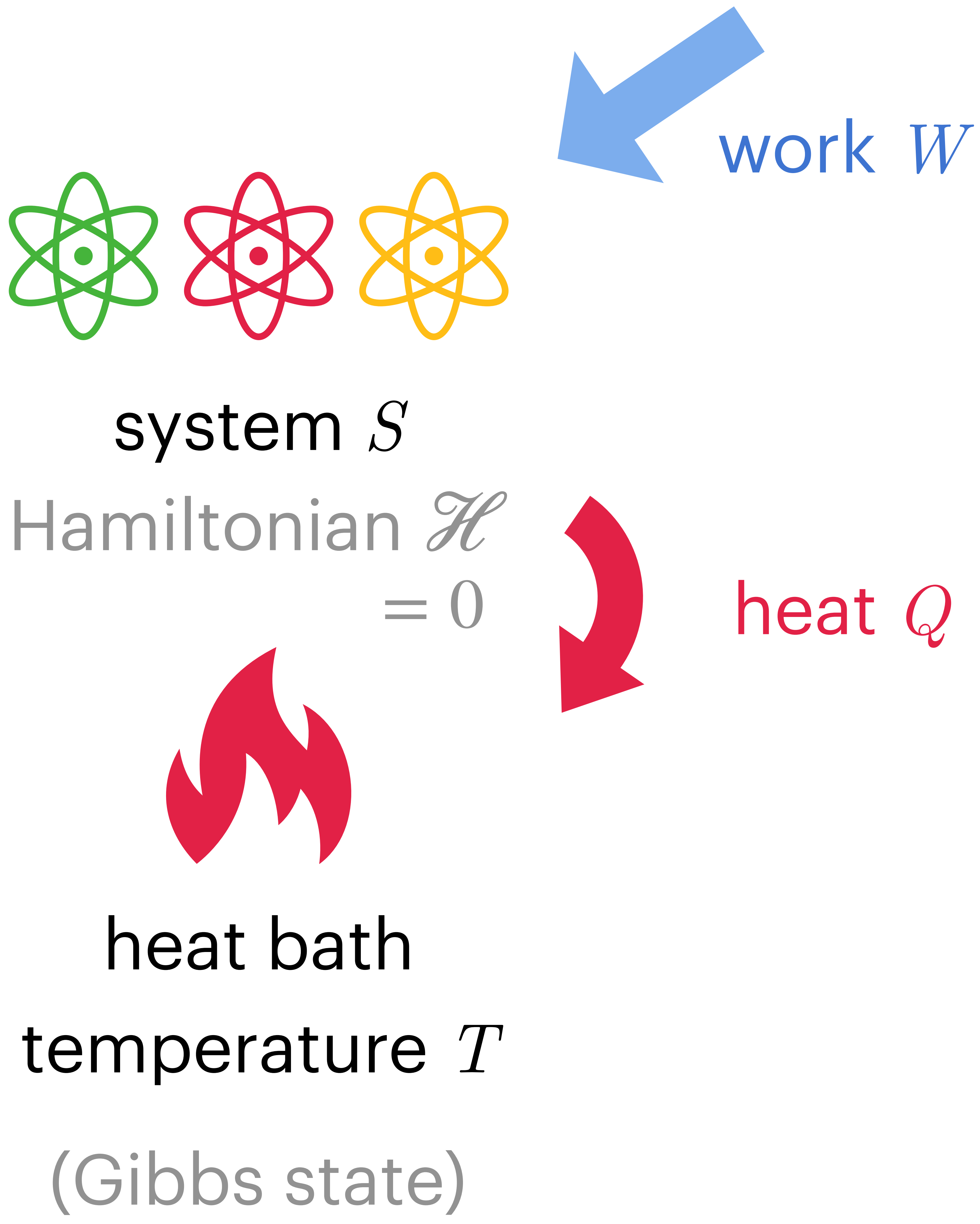


$$W = \Delta E + Q$$

Hamiltonian dependent

irreversibility

Thermodynamics



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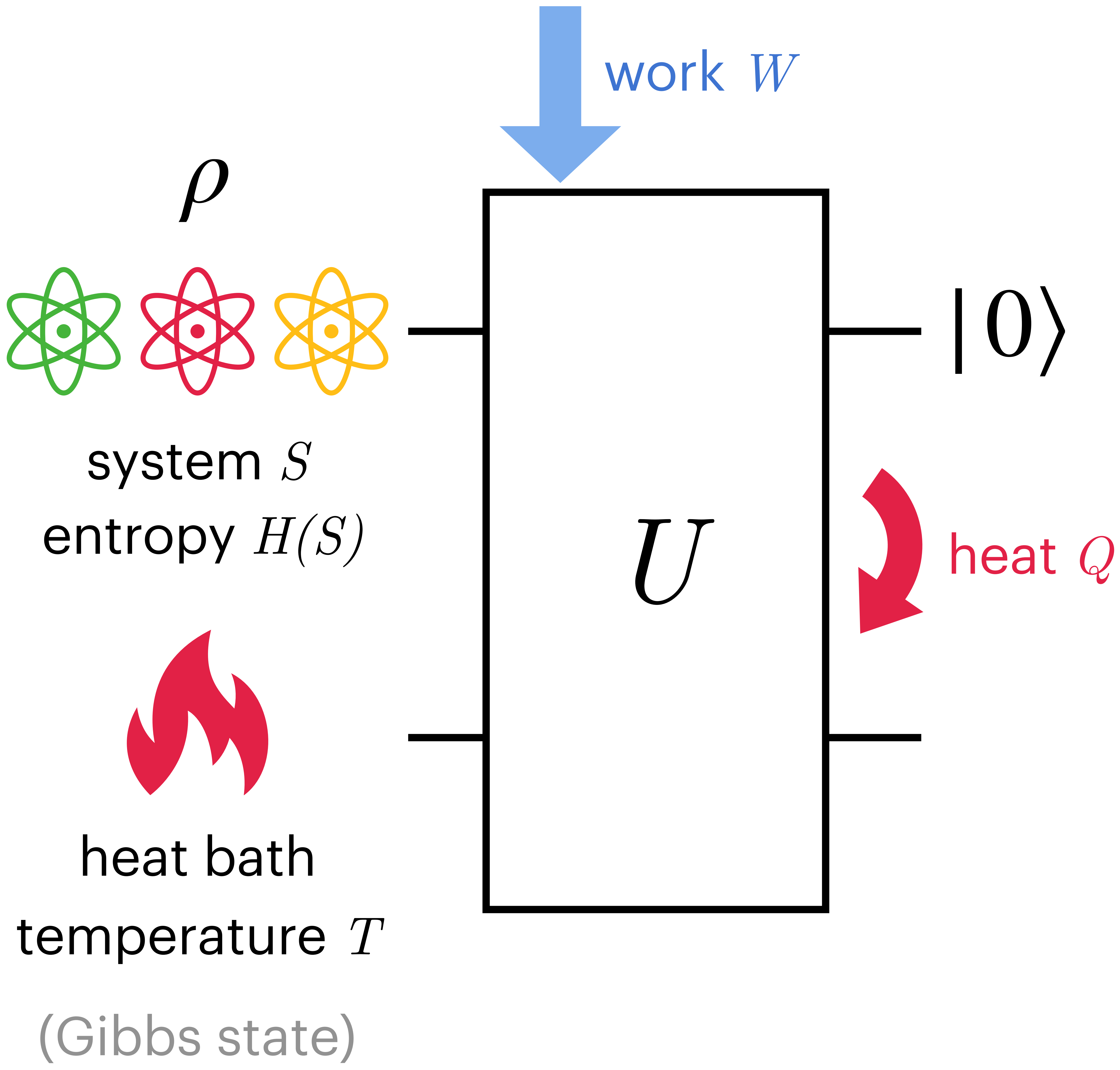
Erasure

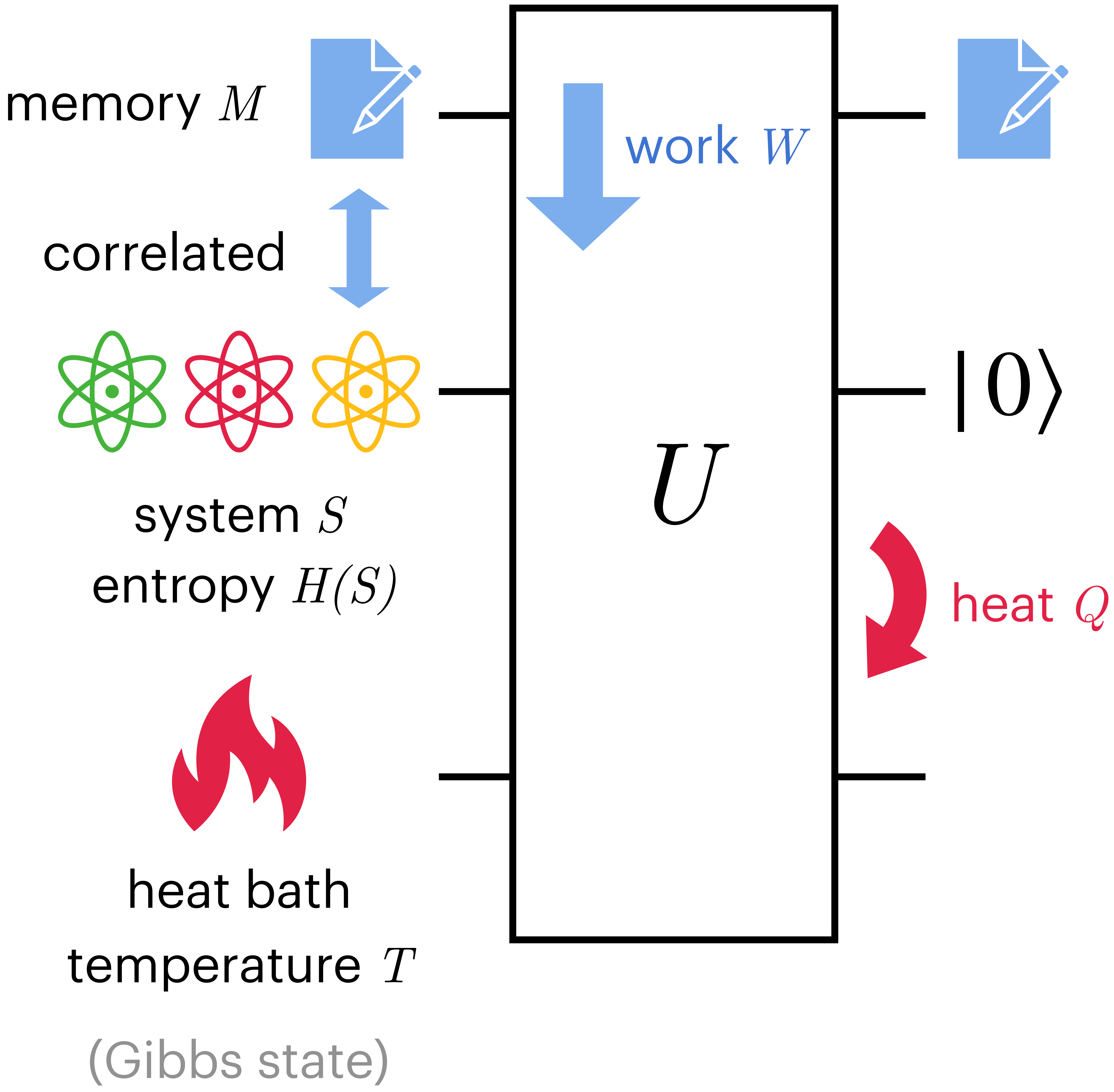


k: Boltzmann constant

$$W = Q = H(S)kT\ln 2$$

Landauer's principle

Reeb & Wolf New Journal of Physics 2014



Erasure with side info

An extreme case: $\rho_{SM} = \sum_x p_x |\psi_x\rangle\langle\psi_x|_S \otimes |x\rangle\langle x|_M$

$H(S|M)=0$

$$W = Q = H(S|M)kT\ln 2$$

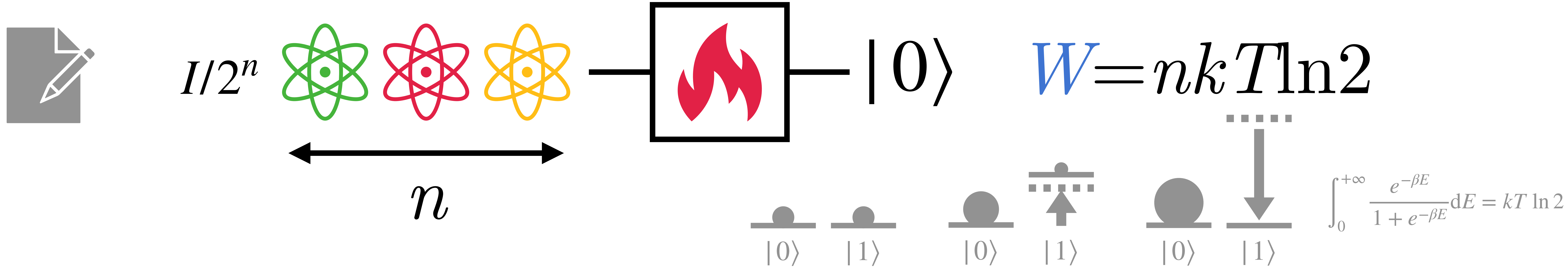
Landauer's principle

For quantum memory, $H(S|M)$ may be negative.

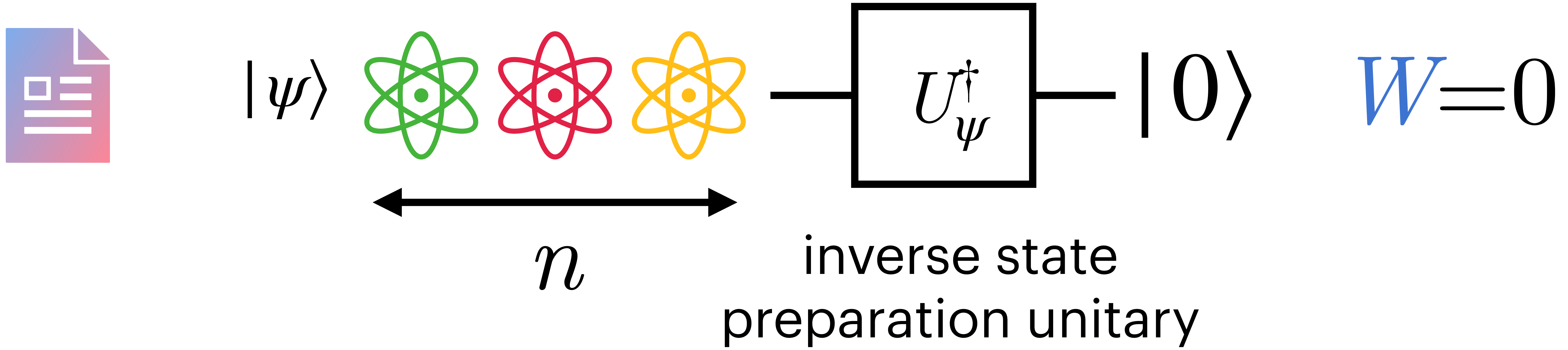
del Rio et al. Nature 2011

Erasure with side information

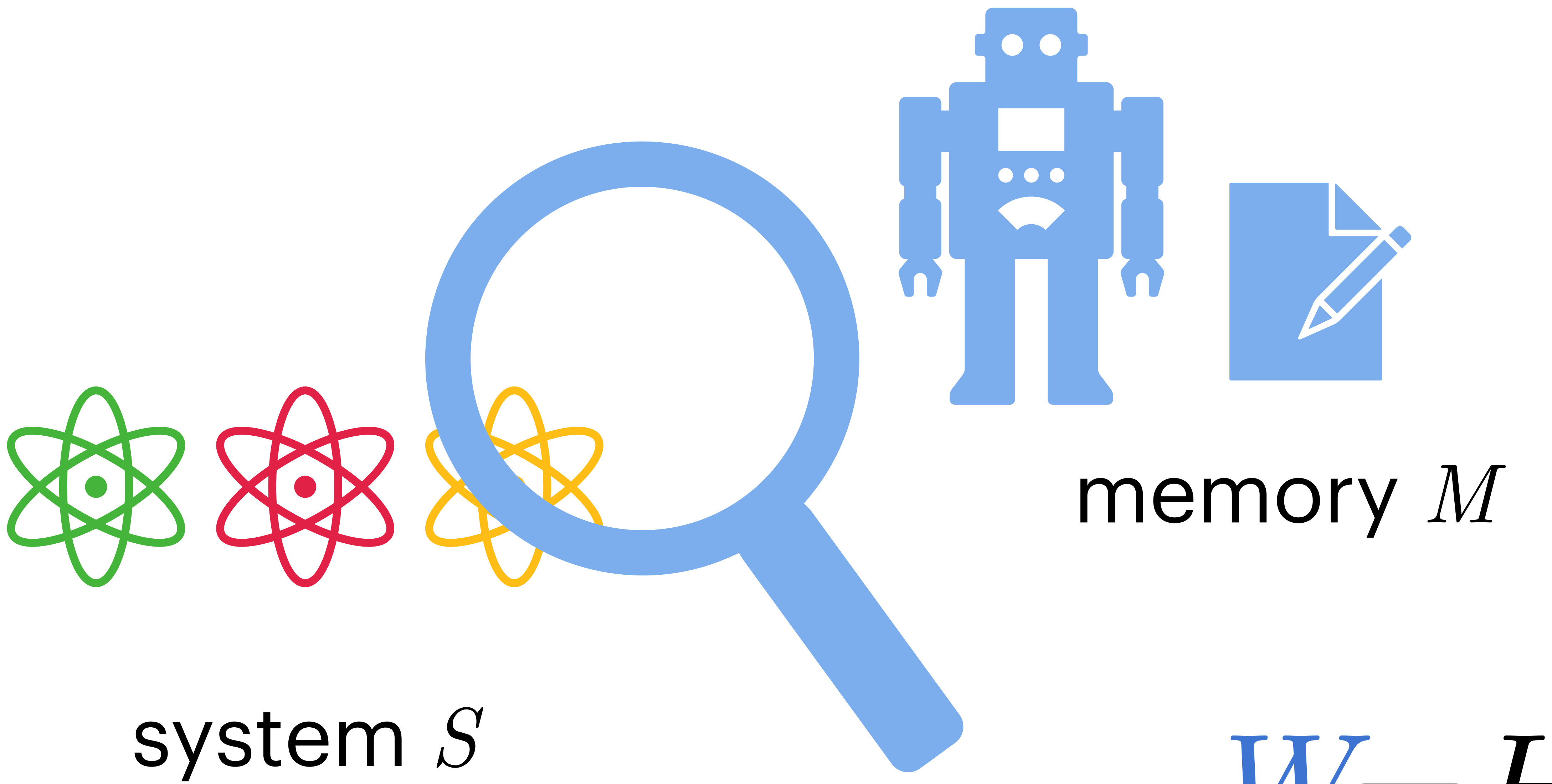
Complete ignorance: $H(S|M)=n$



Complete knowledge: $H(S|M)=0$



Question

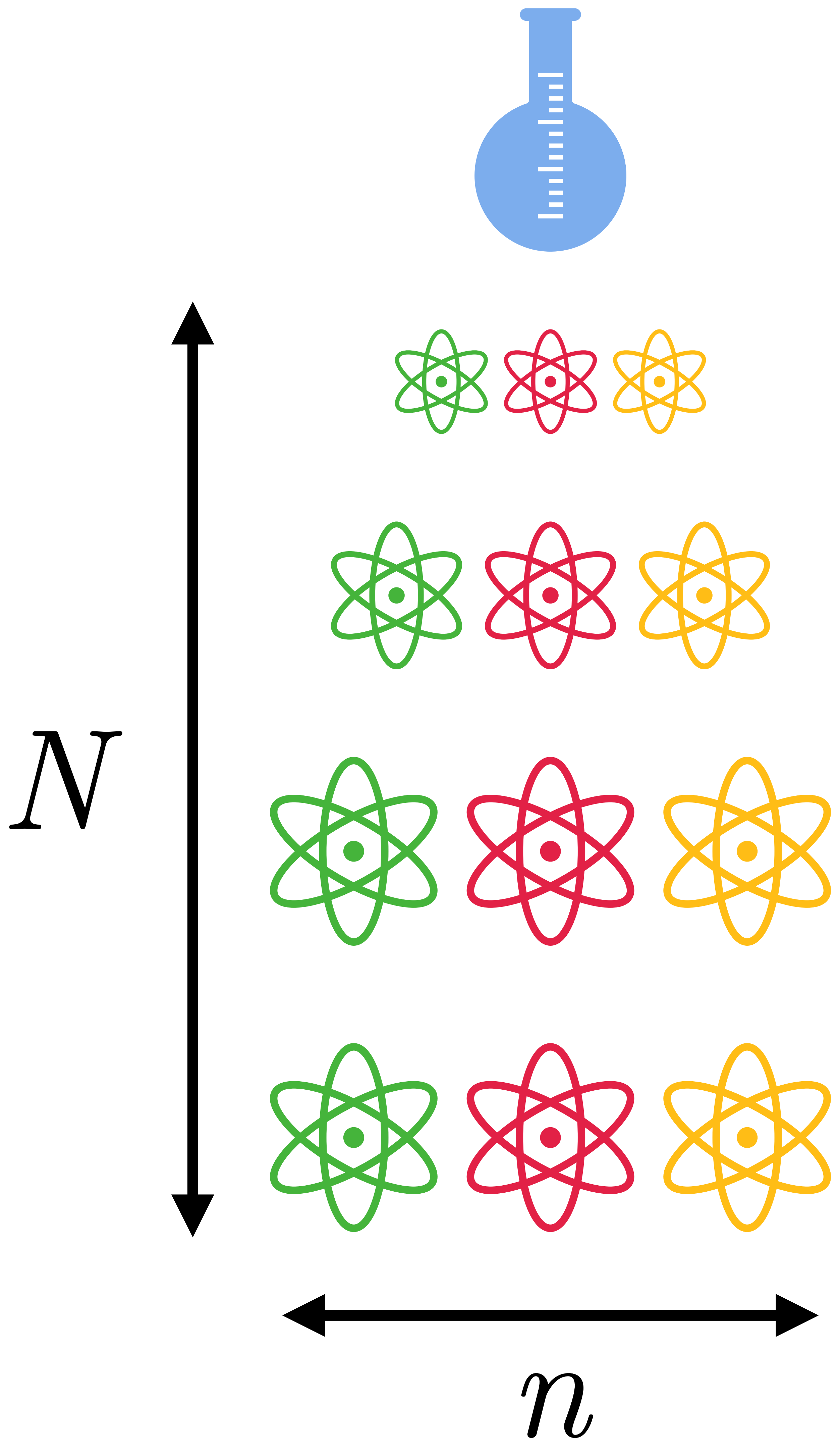


$$W = H(S|M)kT\ln 2$$

How to acquire such knowledge to reduce work cost?

What is the work cost of acquiring such knowledge?

Learning to erase



N copies of an unknown pure state
from m possibilities: $|\psi_x\rangle, x = 1,\ldots,m$ pairwise $\Theta(1)$ apart

$$\rho = \sum_x p_x (|\psi_x\rangle\langle\psi_x|)^{\otimes N}$$

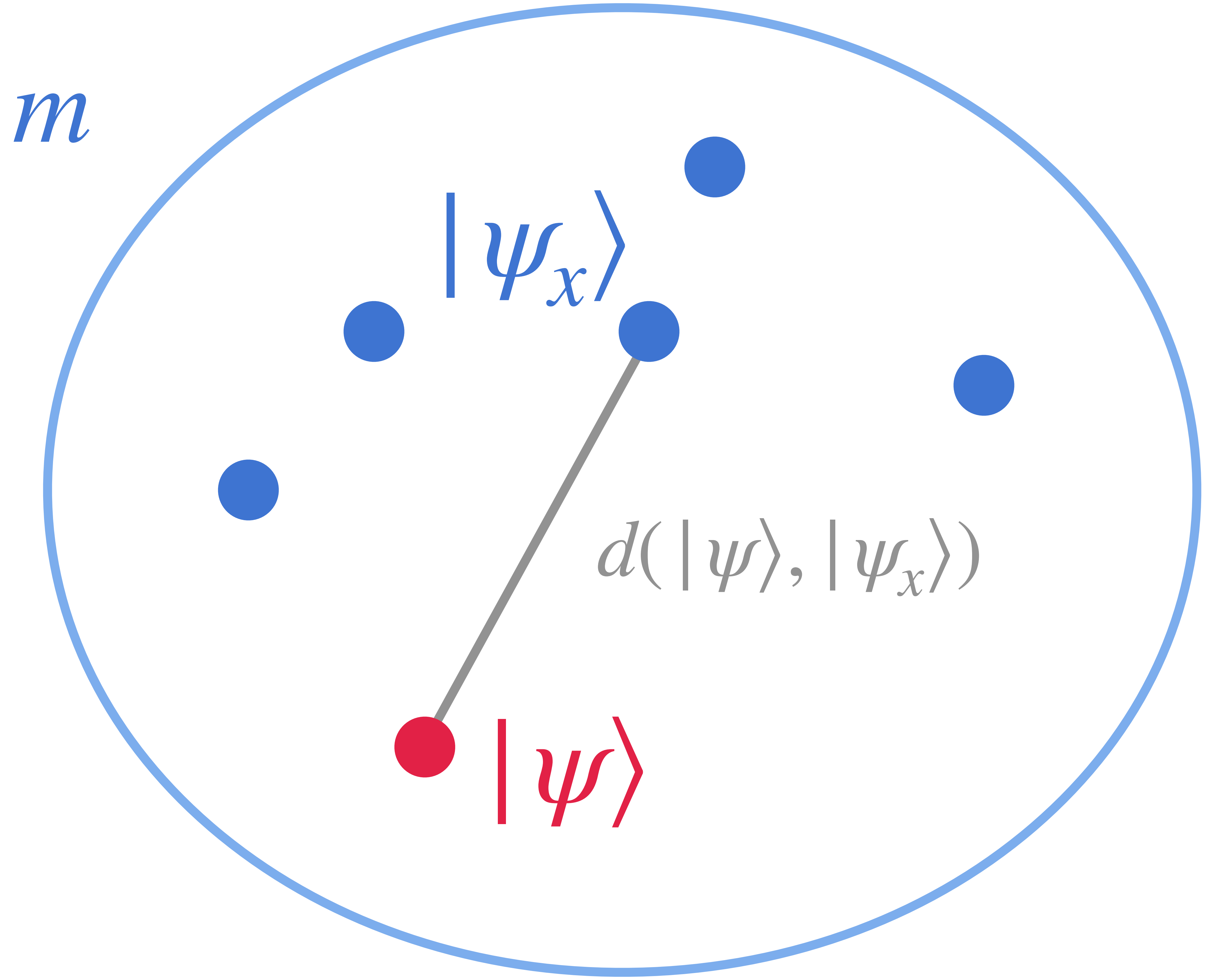
Intuition:

as we collect more copies

ignorance is reduced

=> less work cost per erasure $\frac{W}{N} \to 0$

Learning algorithm



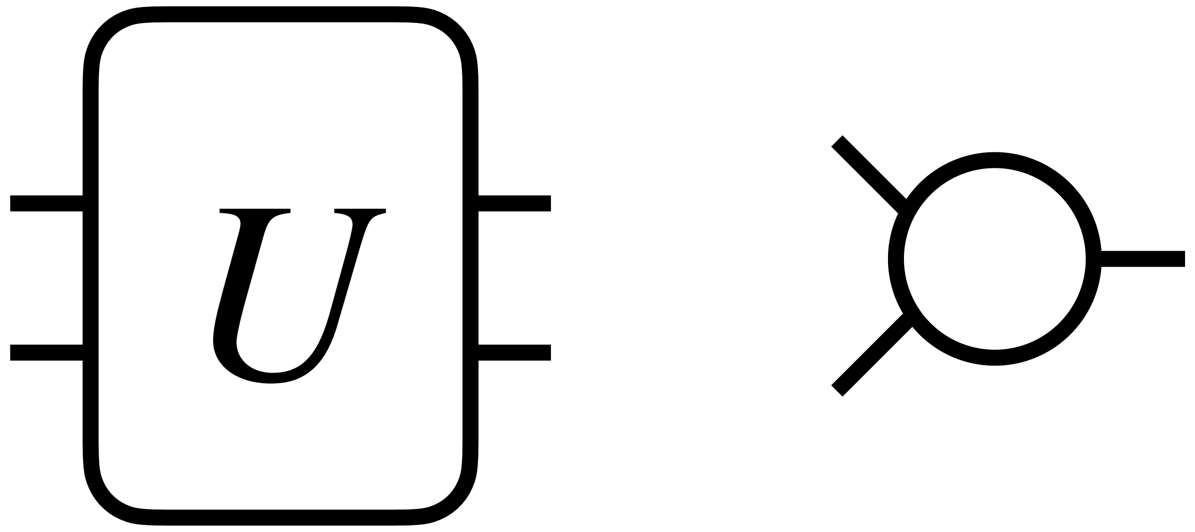
- Estimate the trace distance between target state $|\psi\rangle$ and every candidate $|\psi_x\rangle$ using Clifford classical shadow
- Output the closest one
- Optimal sample complexity $s = O(\log m)$

What is the energy cost?

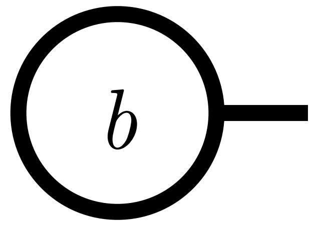
Zhao et al. PRX Quantum 2024, Huang et al. Nature Physics 2020, Badescu et al. TheoretiCS 2024

Reversible learning

Building blocks of a learning algorithm:

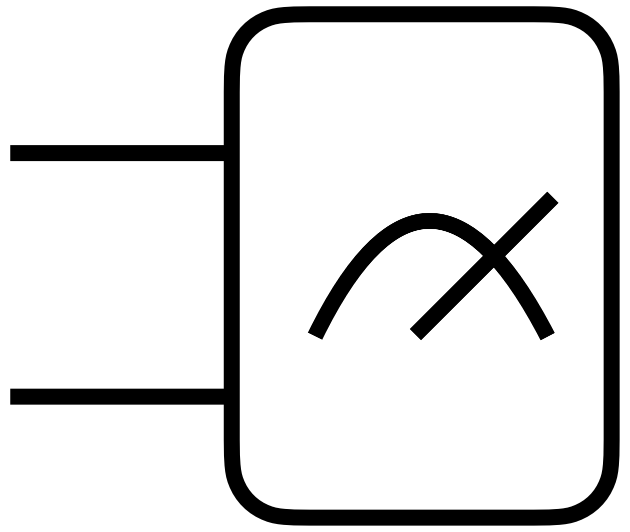


quantum/classical gates



$b \sim \mathrm{Bern}(1/2)$

random number



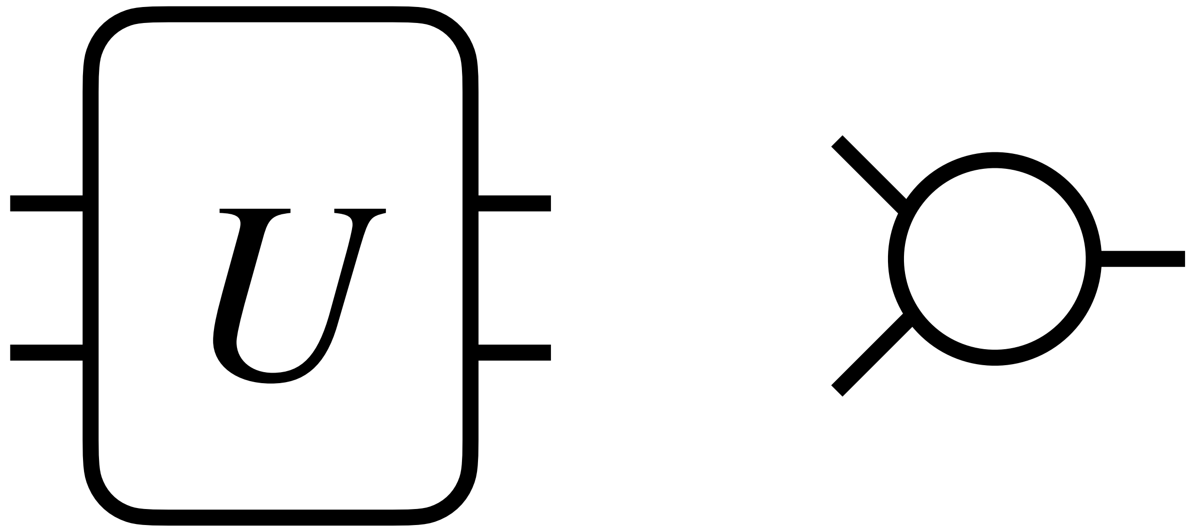
$\{P_a\}$

projectors

measurement

Reversible learning

Building blocks of a learning algorithm:



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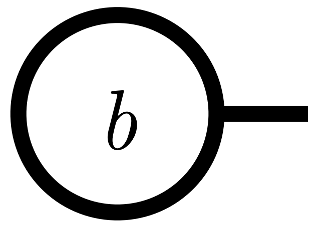

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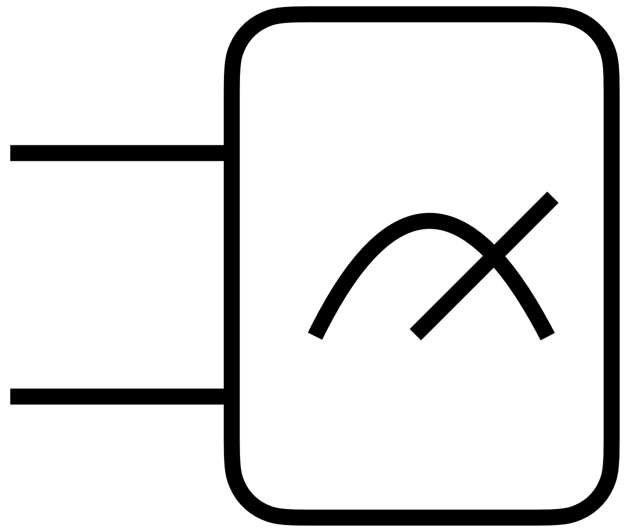

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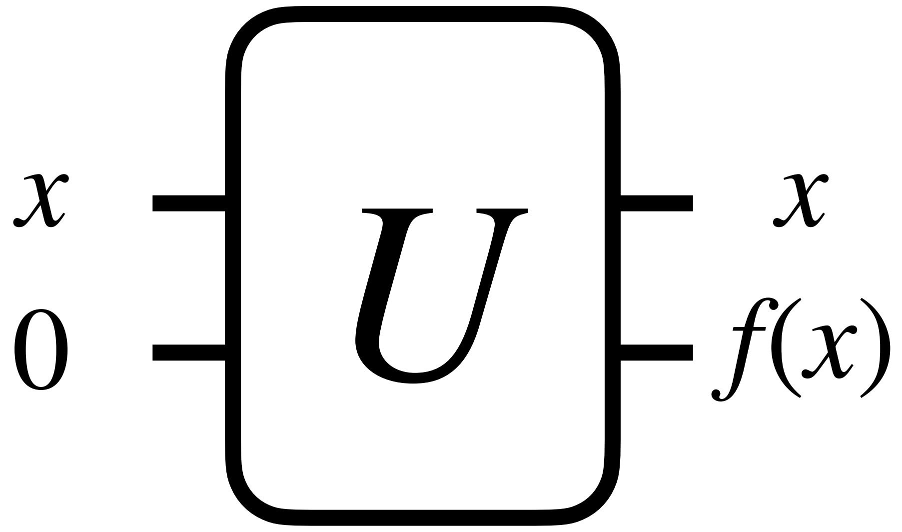
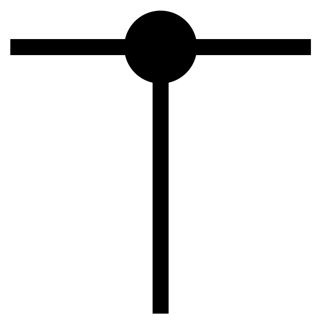
projectors

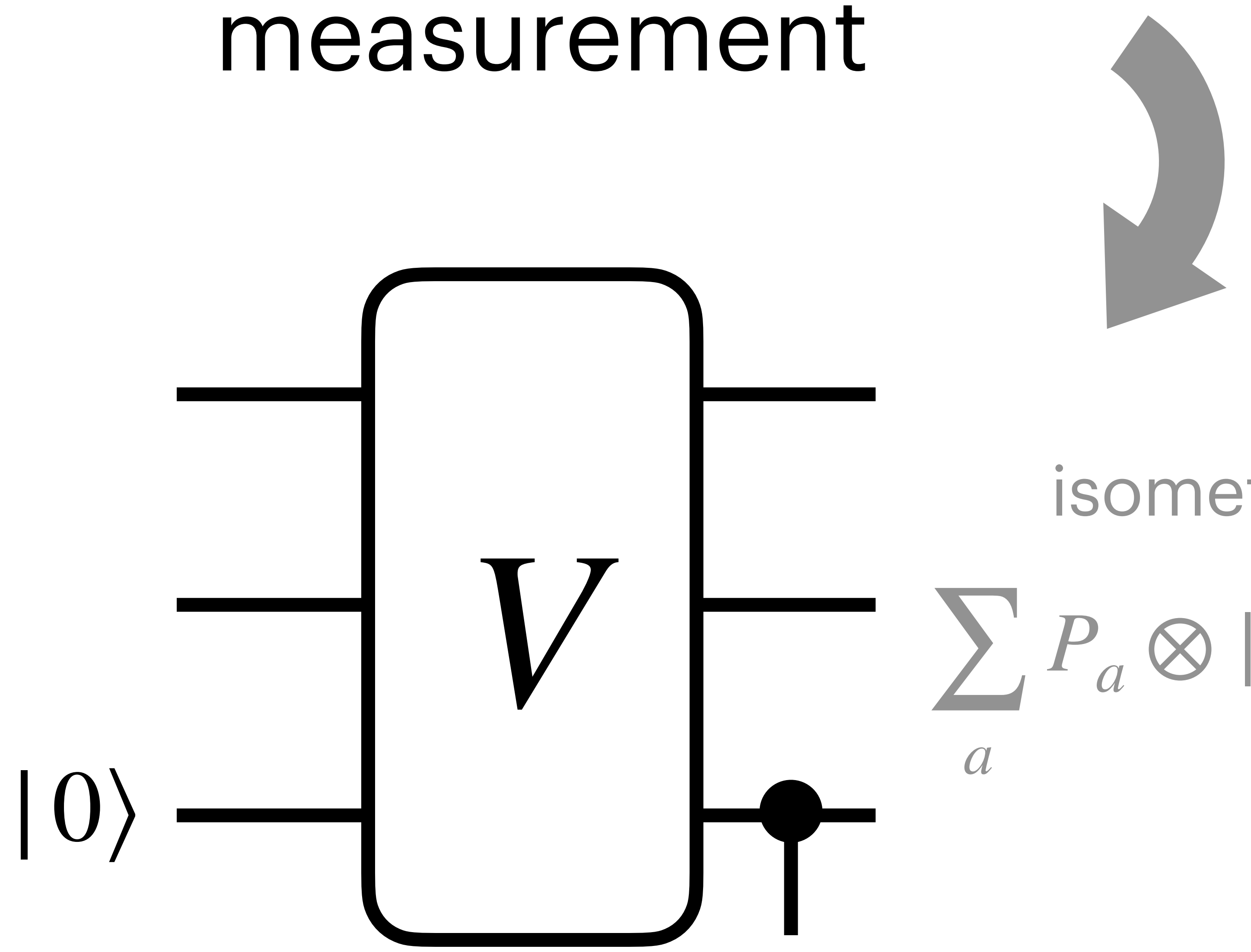
measurement

Make it reversible: fully coherent processing



$|+\rangle$ $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$



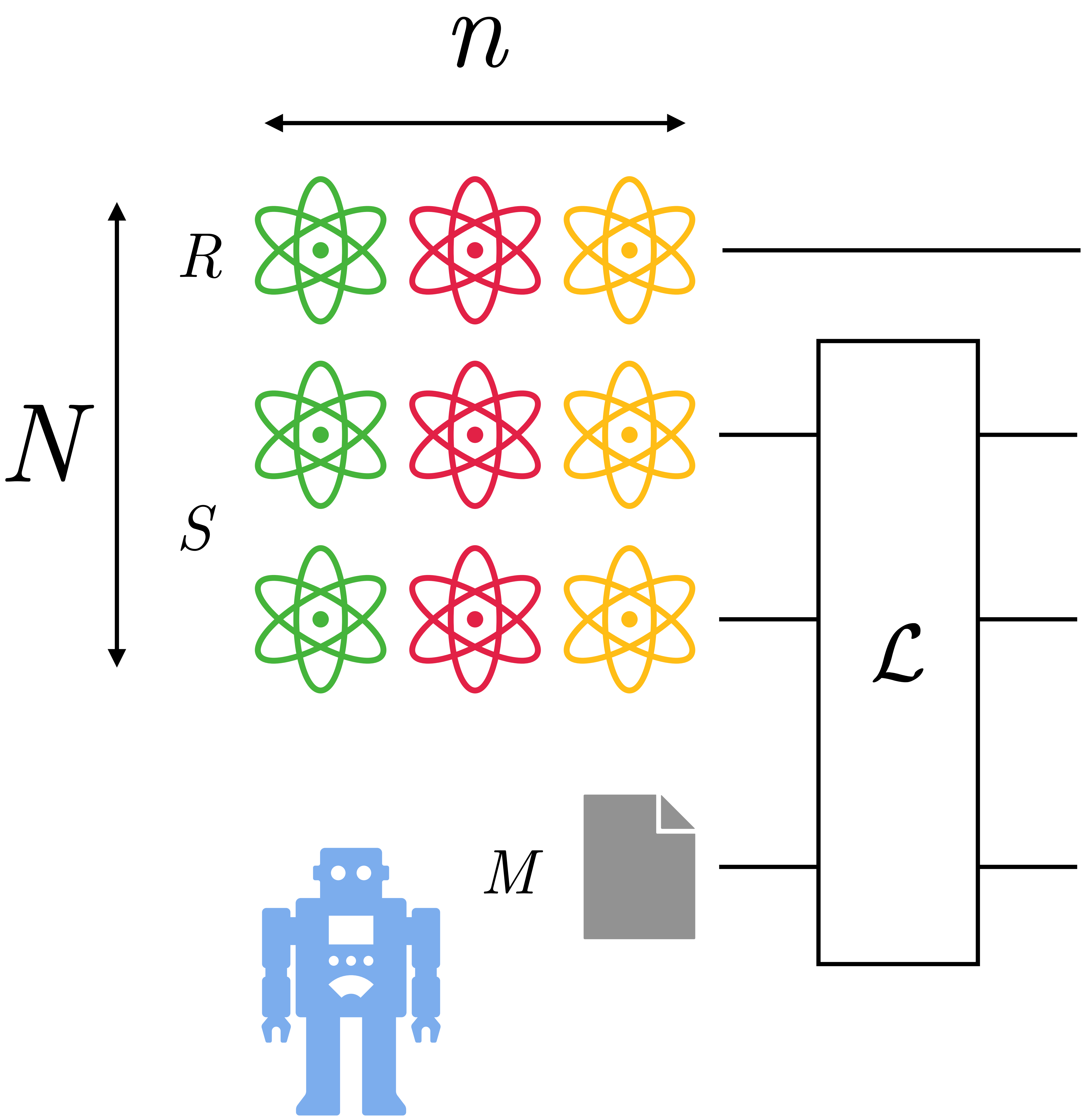


isometry

$\sum_a P_a \otimes |a\rangle\langle 0|$

$$V|\psi\rangle|0\rangle = \sum_a P_a|\psi\rangle|a\rangle$$

Reversible learning

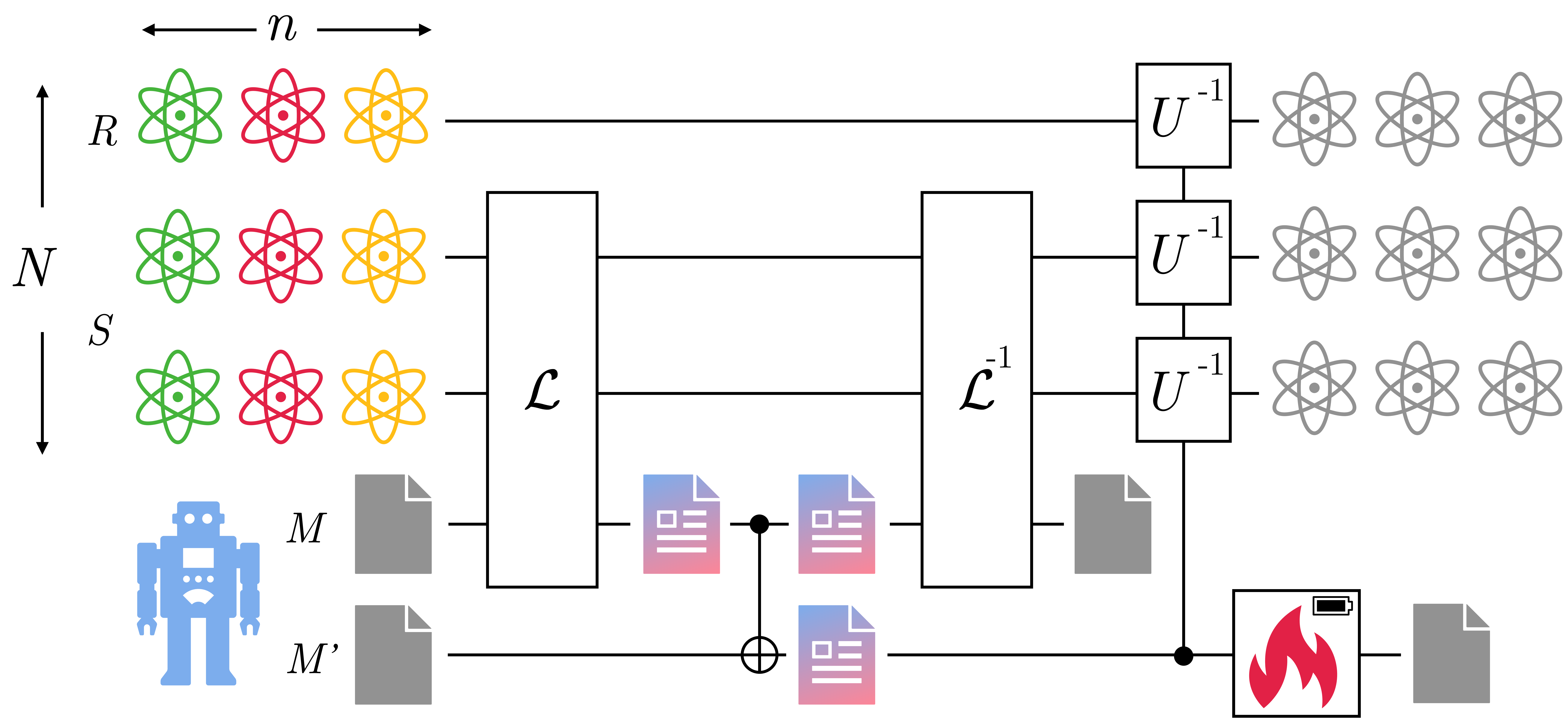


Reversible learning algorithm:

$$\mathcal{L}\,|\psi_x\rangle_S^{\otimes s}\,|0\rangle_M\,|0\rangle_A = \sum_{x'=1}^{m} c_{x'|x}\,|x'\rangle_M\,|\mathrm{junk}_{x'}\rangle_{S,A}$$

Learning guarantee: $|c_{x|x}|^2 \geq p_{\mathrm{succ}} \to 1$

Erasure protocol



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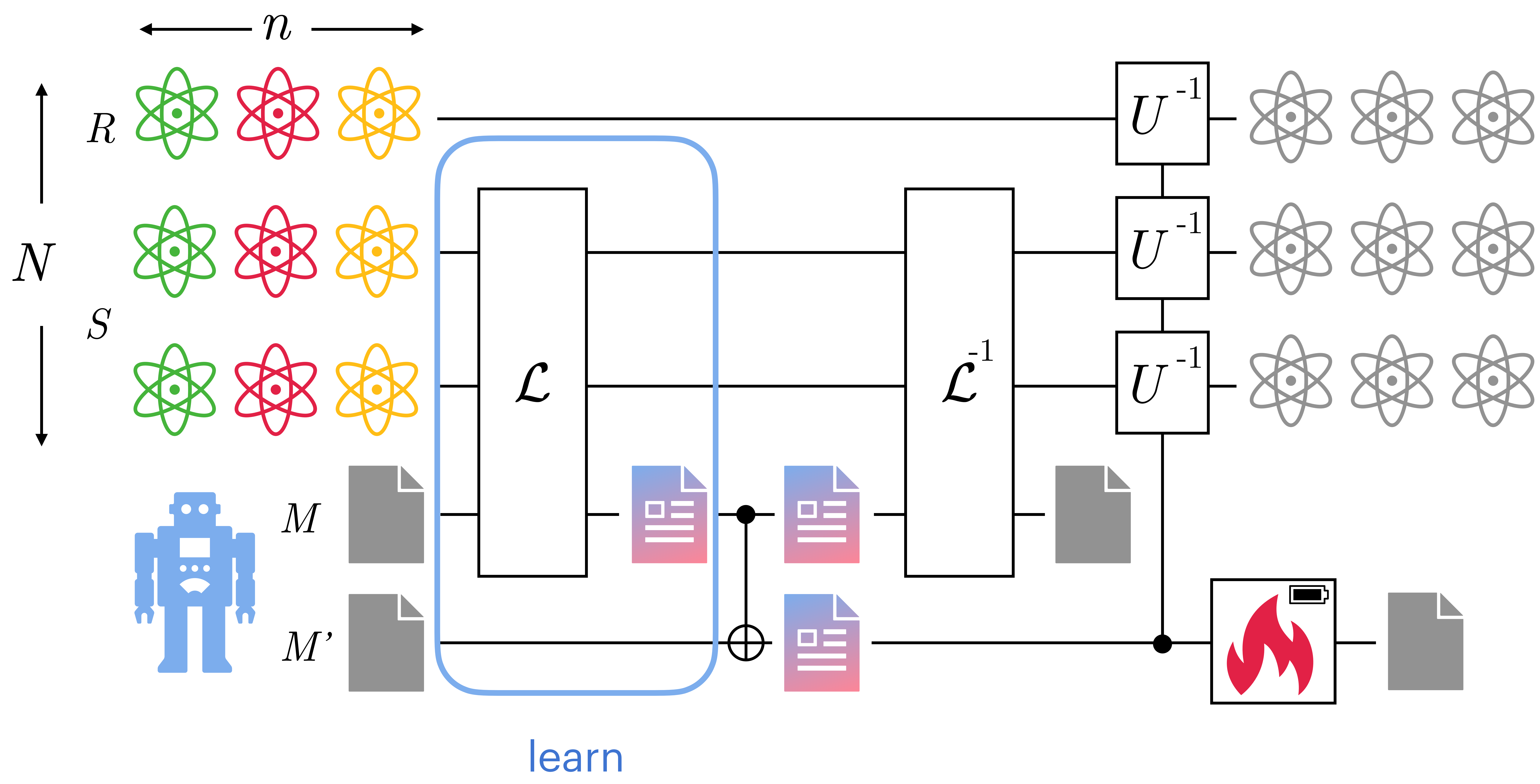

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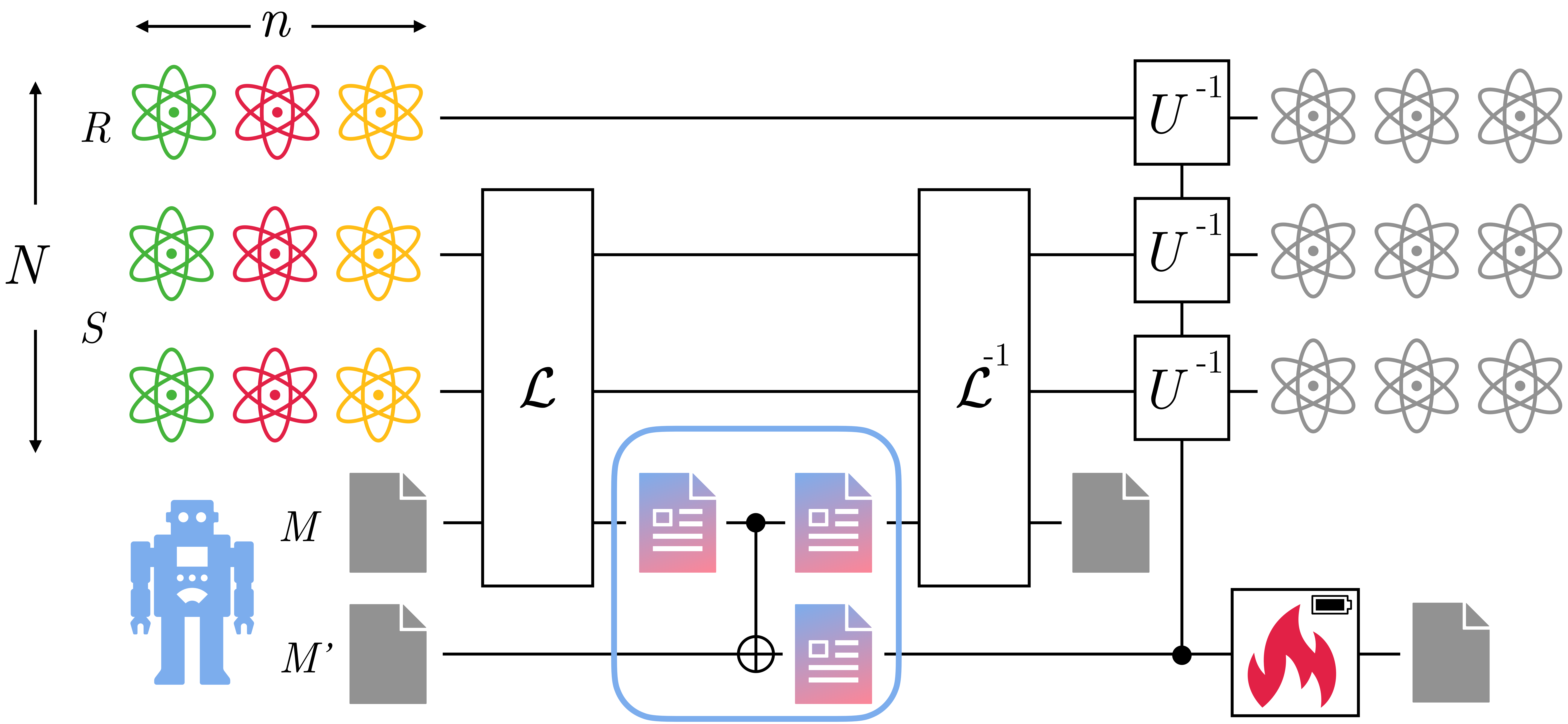
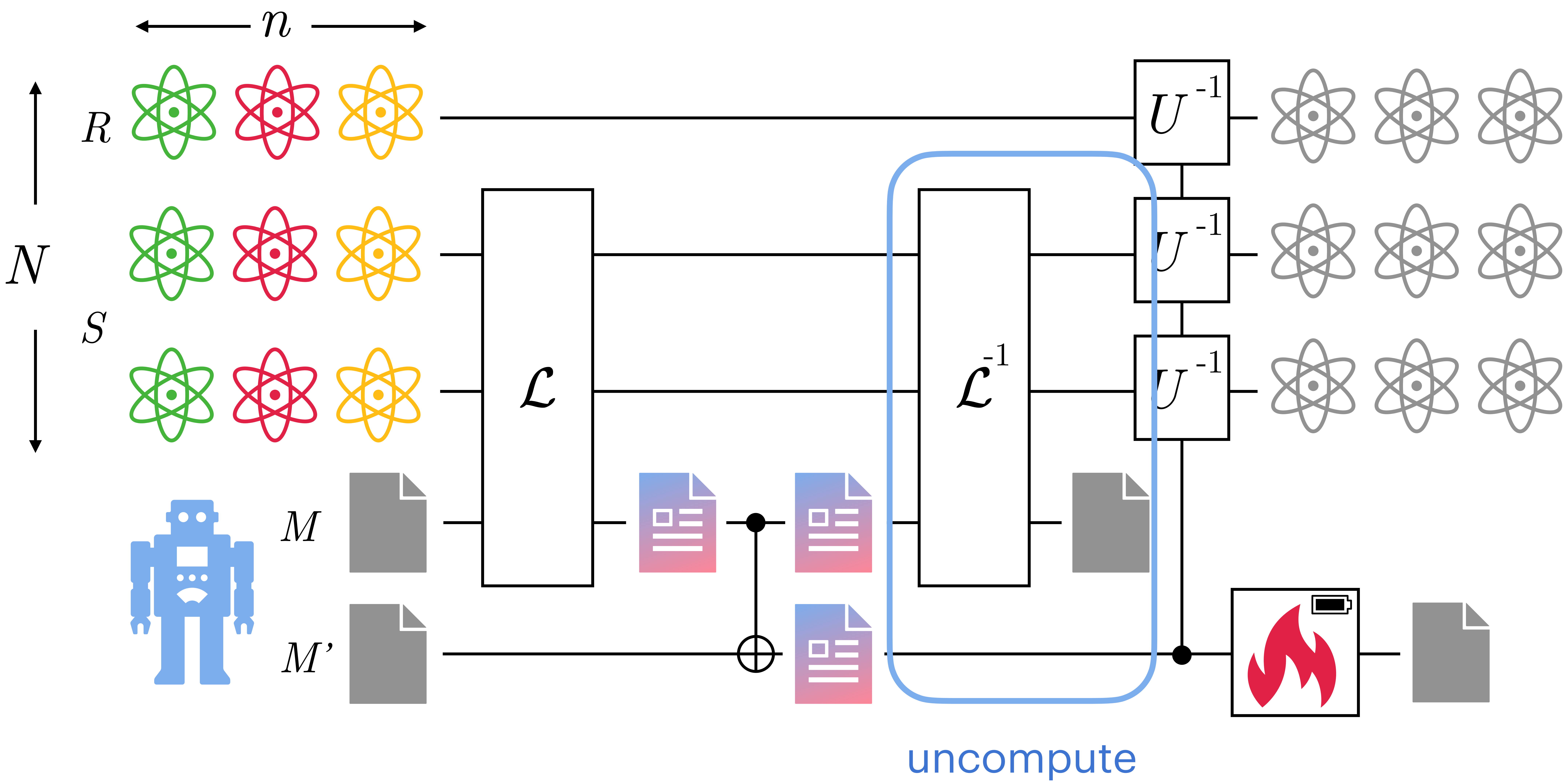


copy out learning outcome

Erasure protocol



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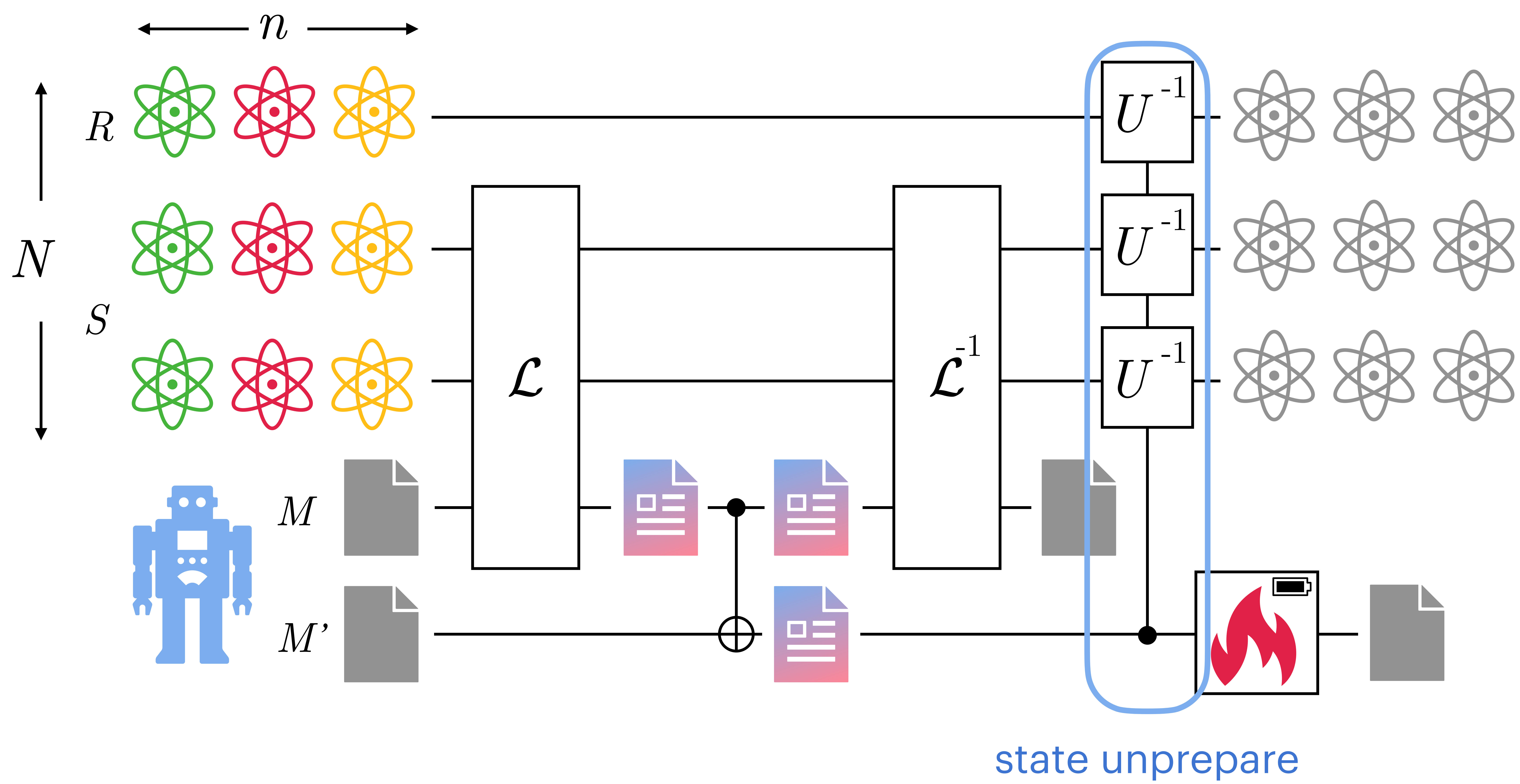

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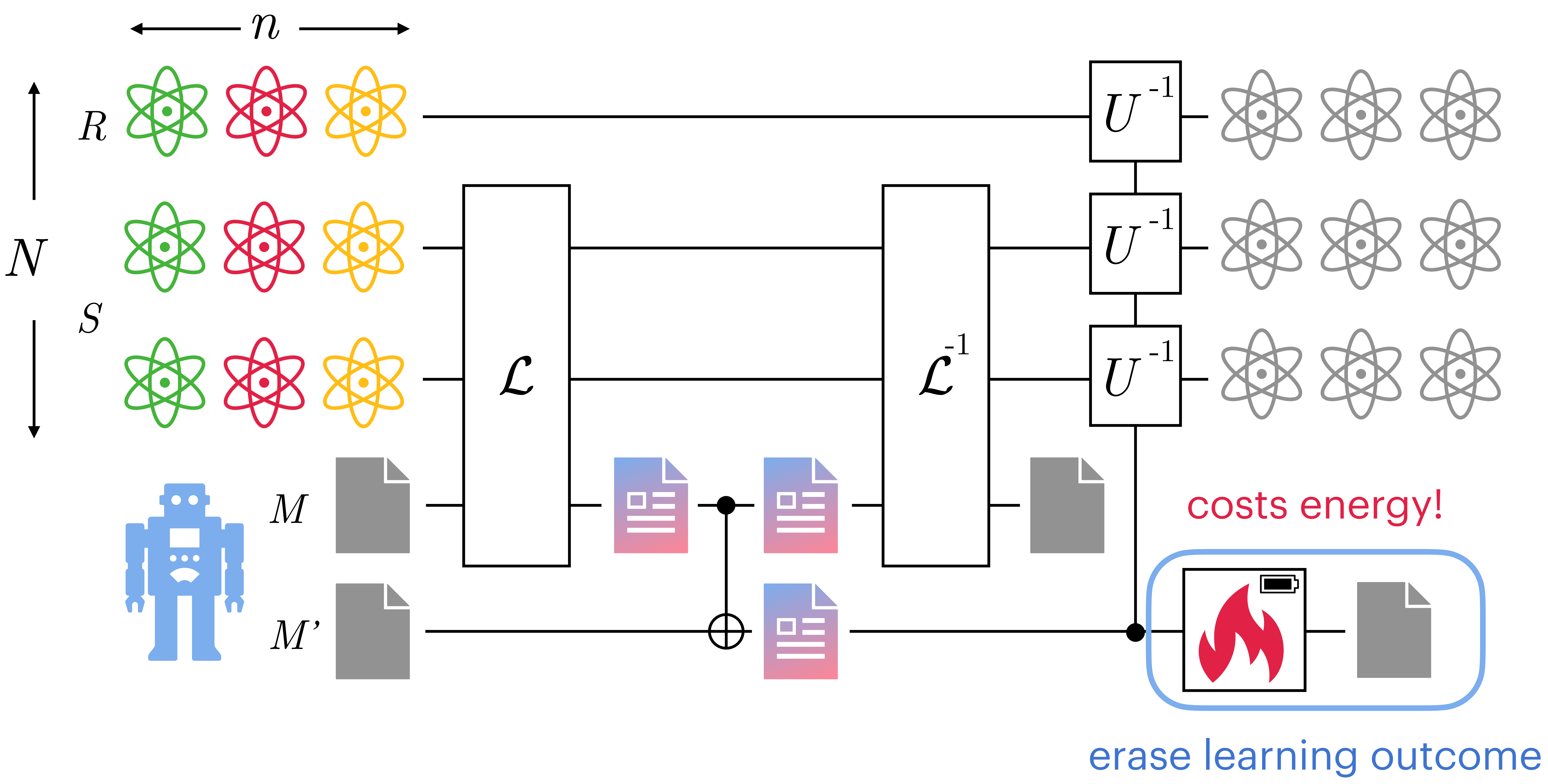

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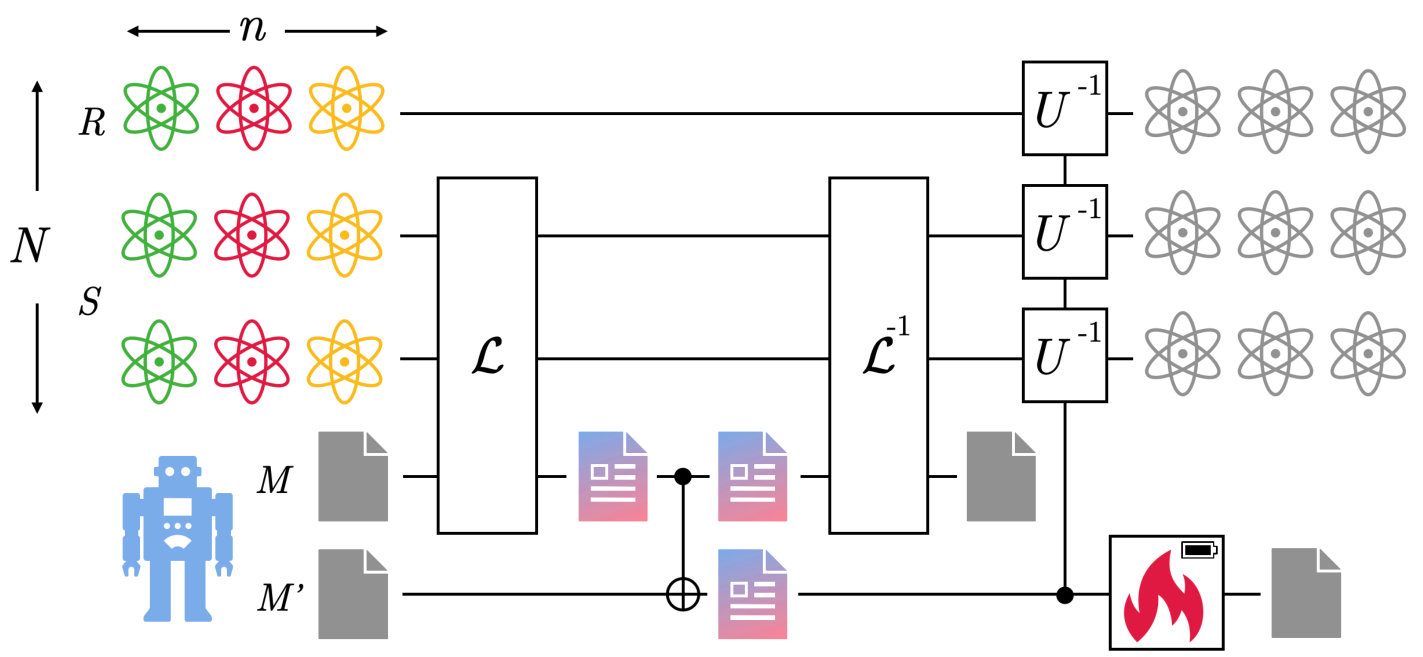


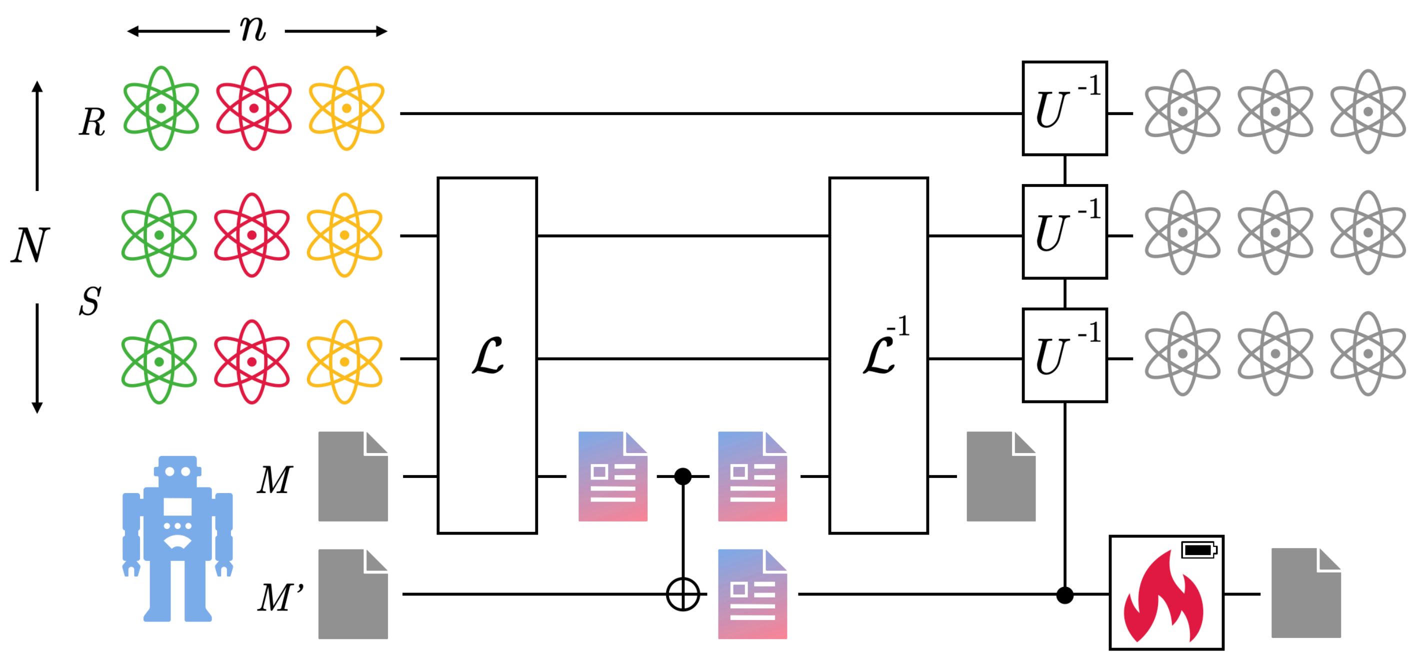
Remarks:

1. Correctness: trace distance from $|0\rangle$ bounded by $\sqrt{1-p_{\text{succ}}^2} \to 0$

#bits to store learning outcome

2. Energy cost: $W = (\log_2 m)kT\ln 2$, independent of N

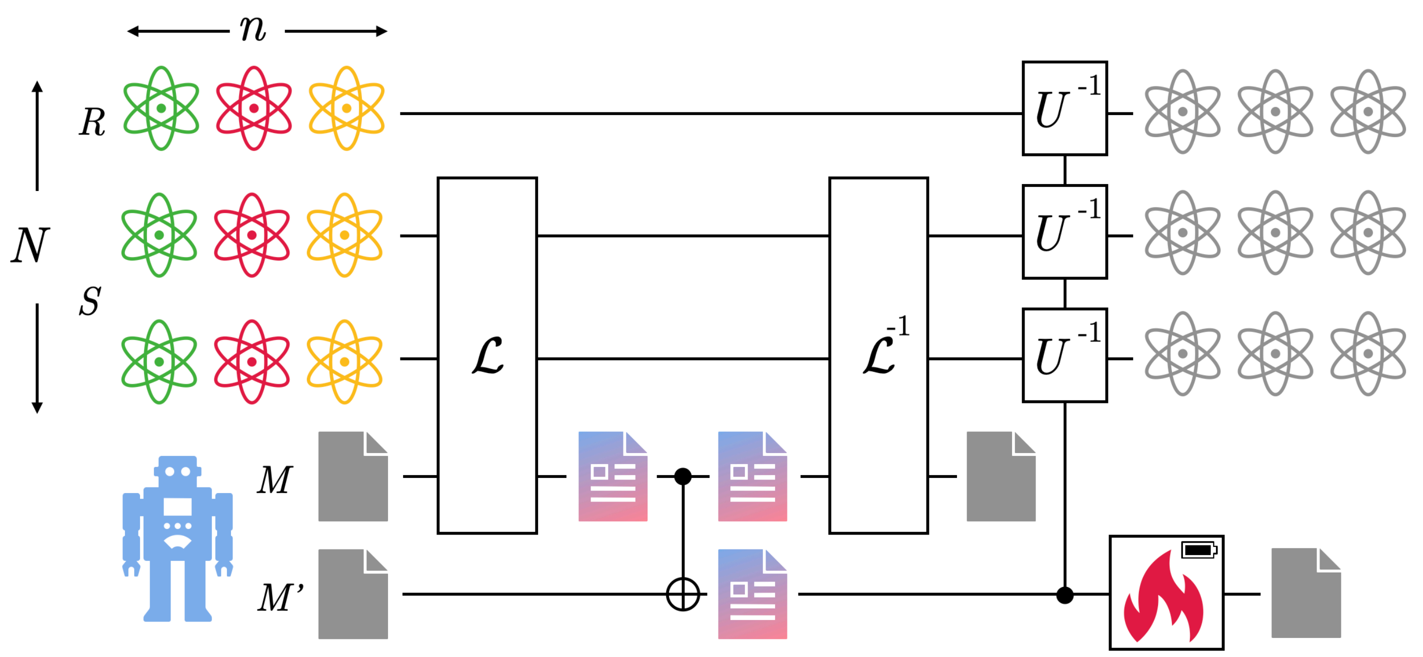
Erasure protocol



Remarks:

1. Correctness: trace distance from $|0\rangle$ bounded by $\sqrt{1 - p_{\text{succ}}^2} \to 0$
2. Energy cost: $W = (\log_2 m)kT \ln 2$, independent of N (#bits to store learning outcome)
3. Learning can be reversible and has no fundamental energy cost itself!
4. The energy cost occurs when we erase the learning outcome.

Erasure protocol



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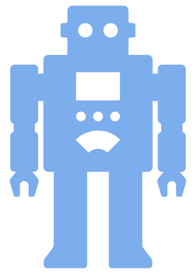
#bits to store learning outcome

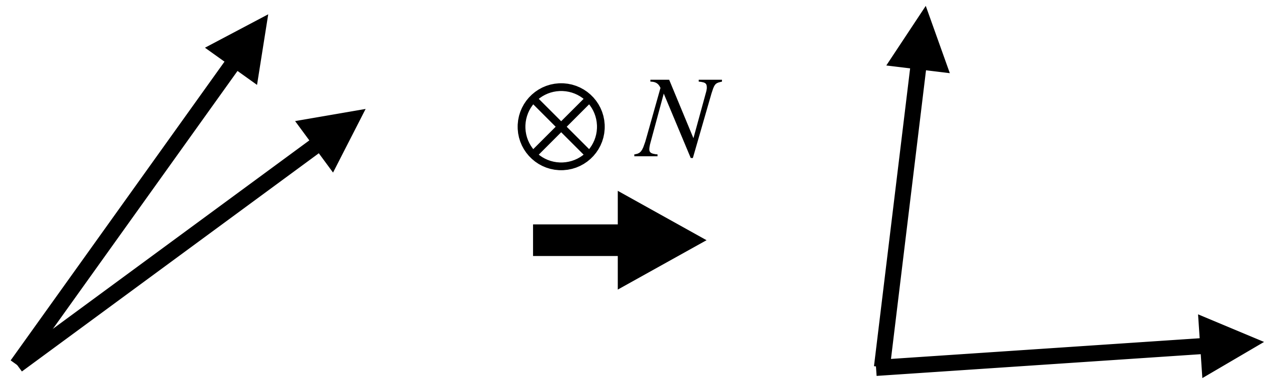
2. Energy cost: $W = (\log_2 m)kT \ln 2$, independent of N
3. Learning can be reversible and has no fundamental energy cost itself!
4. The energy cost occurs when we erase the learning outcome.
5. Sample complexity => minimal quantum memory requirement
6. Time complexity: $O(T_{\text{learn}} + \log m + NT_{\text{prep}})$
7. Efficient learning & state preparation => efficient erasure

Energy optimality

Energy cost of learning to erase: $W = (\log_2 m)kT\ln 2$ 

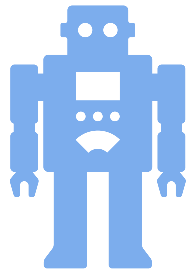
(One-shot) Landauer's limit: $W \geq H_{\max}(\rho)kT\ln 2$ $\quad H_{\max}(\rho) = \log_2 \mathrm{rank}(\rho)$

$$\rho = \sum_{x=1}^{m} p_x(|\psi_x\rangle\langle\psi_x|)^{\otimes N}$$



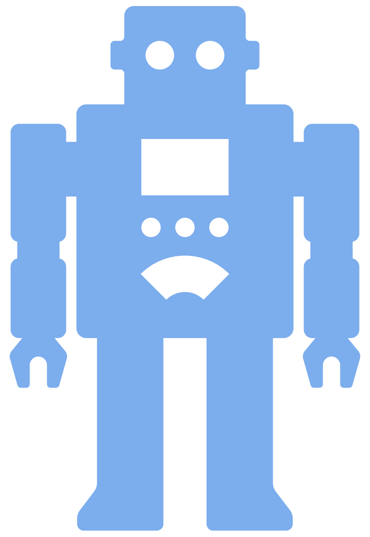
(sample complexity)

When $N \geq \Omega(\log m)$, Gram matrix $G_{x,x'} = \langle\psi_x|\psi_{x'}\rangle^N$ is diagonally dominant

=> $\mathrm{rank}(\rho) = m$ and Landauer's limit coincides with $W = (\log_2 m)kT\ln 2$ 

Dahlsten et al. New Journal of Physics 2011

Efficiently erasable states



Energy cost of learning to erase: $W = (\log_2 m) kT \ln 2$

Erasure is efficient when learning & state preparation is efficient.

efficient: time is polynomial in n, N

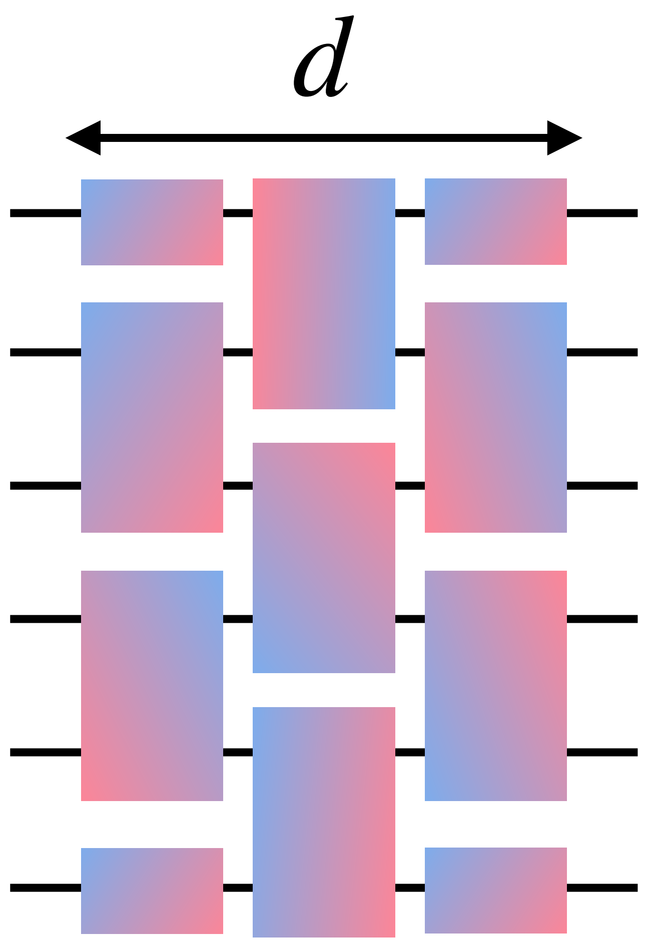
inefficient: for $n \sim 100$, may take longer than the age of the universe

Physically relevant examples:

short time dynamics

NISQ

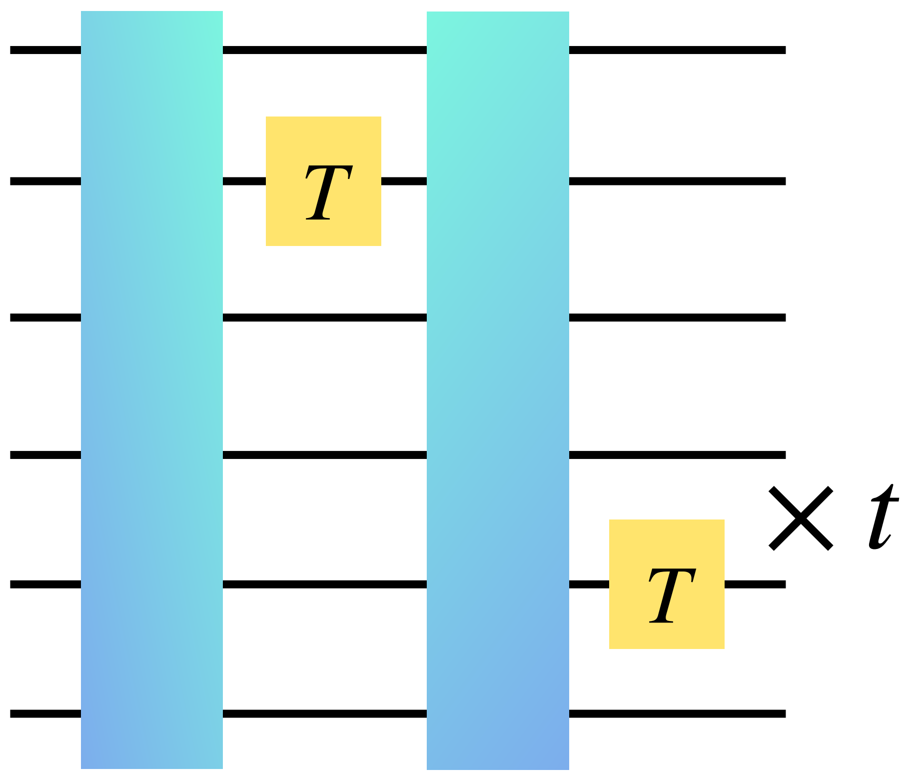
quantum advantage



shallow circuit states

low complexity

QEC



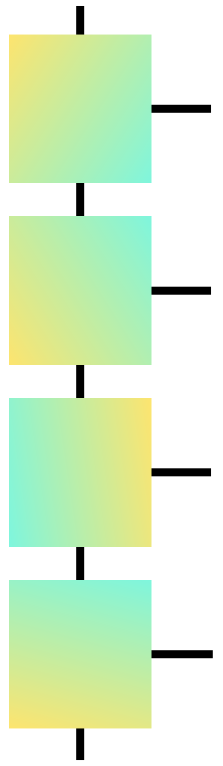
doped stabilizer states

low magic

ground state

SPT

MBQC



$\log_2 \chi \leq \mathcal{S}$

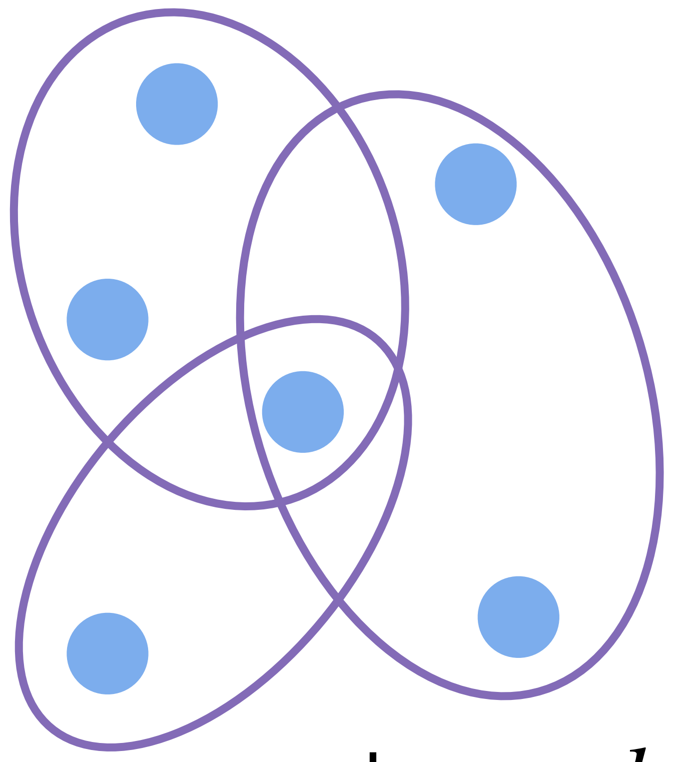
matrix product states

low entanglement

query algorithm

MBQC

IQP

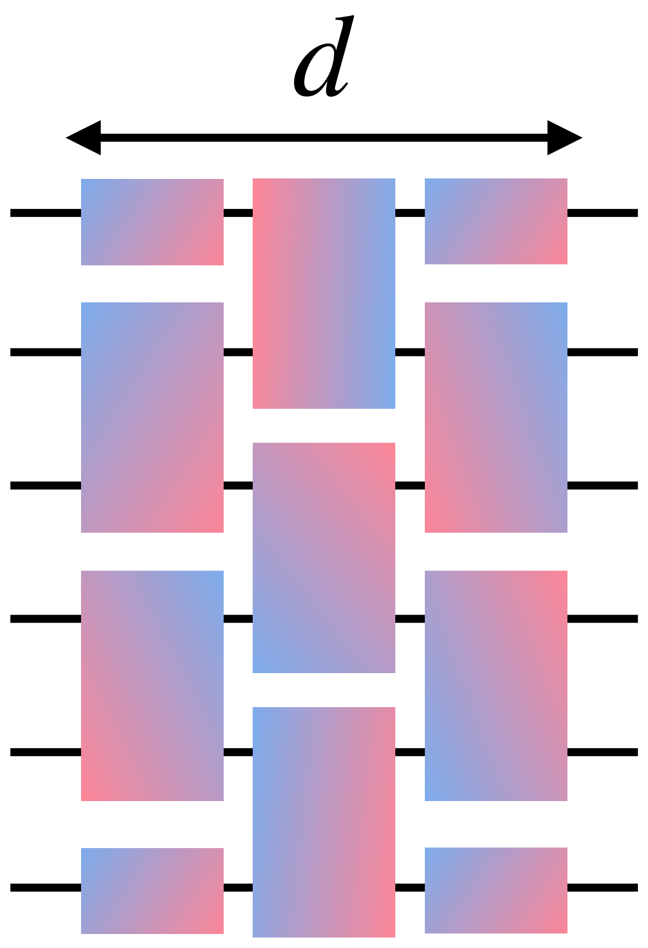


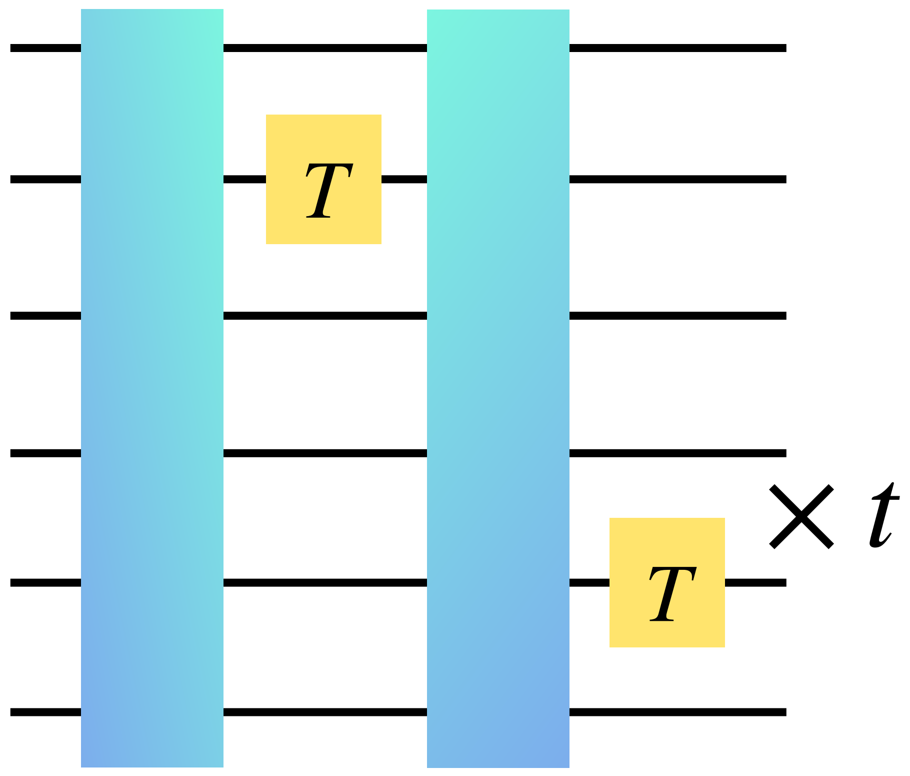
degree k

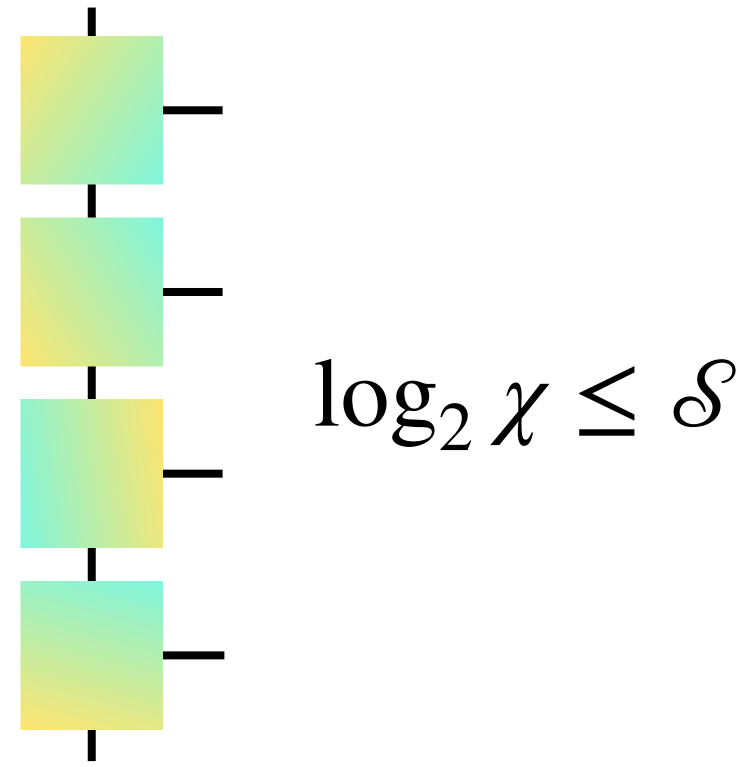
phase states

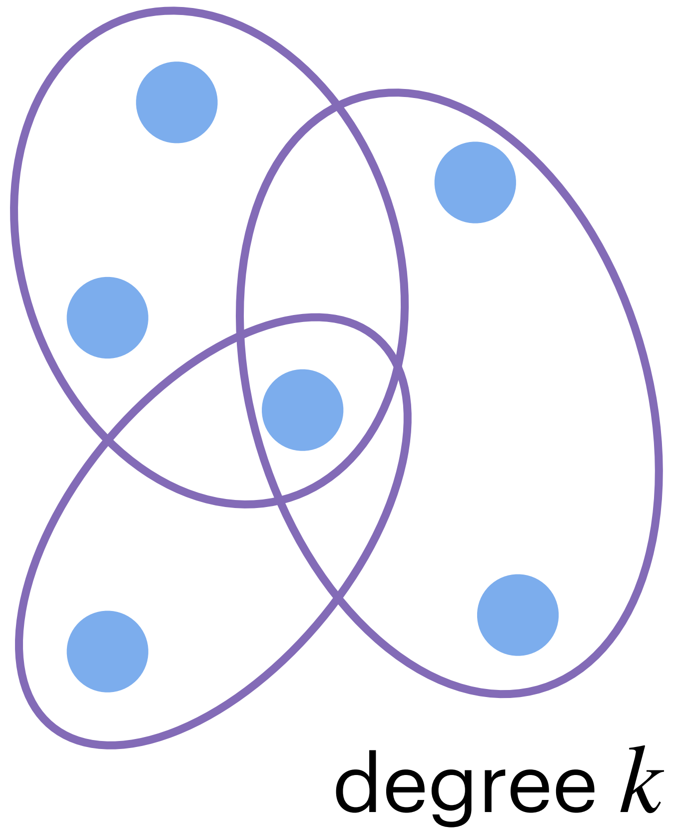
low degree

Work vs complexity





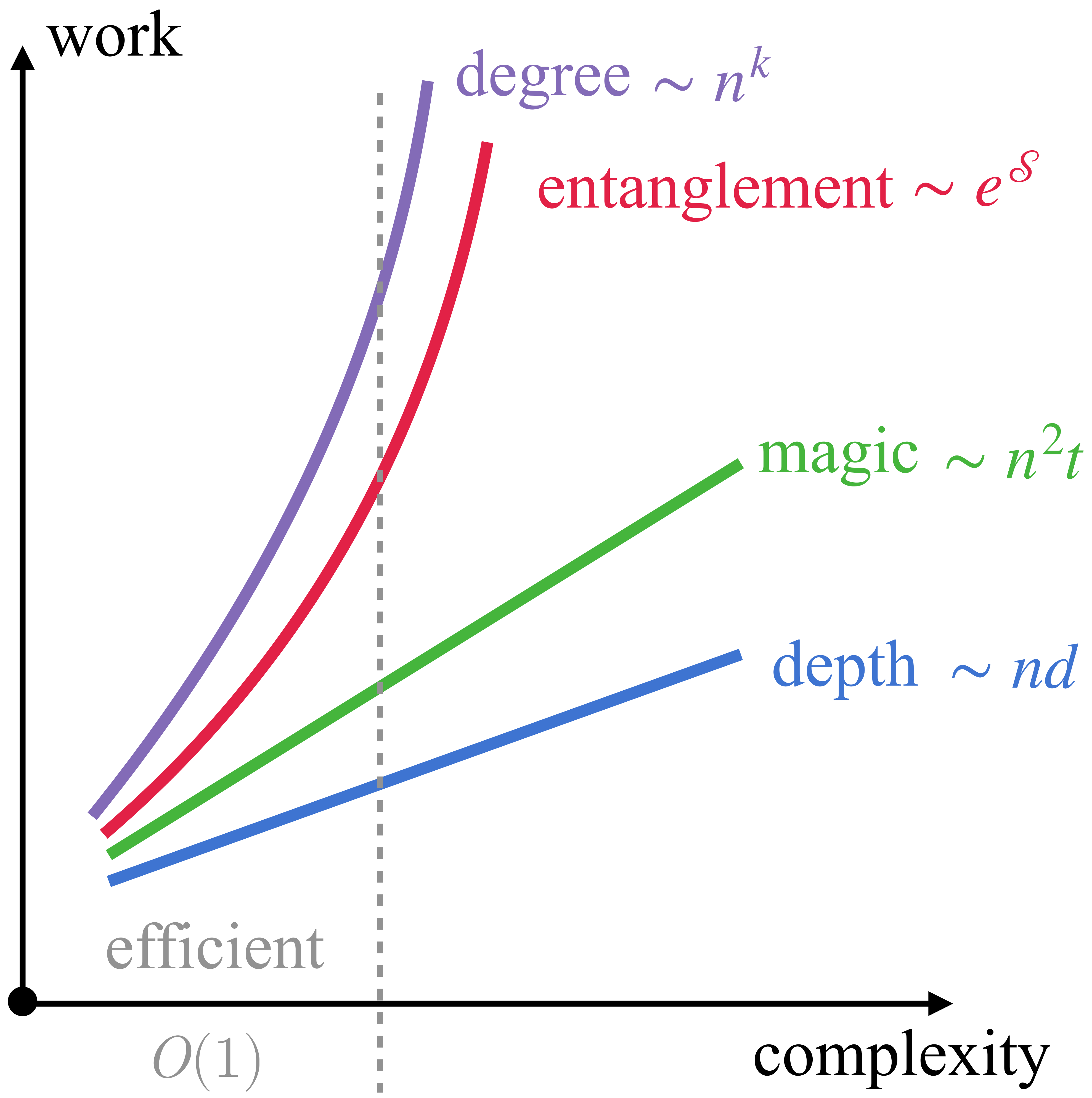




	shallow circuit states	doped stabilizer states	matrix product states	phase states
complexity	depth d	t T-gate	entanglement entropy $\mathcal{S}$	degree k
work cost	$\Theta(nd)kT\ln 2$ gate count	$\Theta(n^2 t)kT\ln 2$ Clifford has gate count $O(n^2)$	$\exp(\tilde{\Theta}(\mathcal{S}))kT\ln 2$ continuous class needs covering/packing	$\Theta(n^k)kT\ln 2$ #polynomial with degree k
time complexity	$O(\mathrm{poly}(n)2^{d^{O(1)}} + ndN)$ local inversion	$O(\mathrm{poly}(n,2^t) + n^2 tN)$ Bell sampling	$O(\mathrm{poly}(n,2^{\mathcal{S}}) + n4^{\mathcal{S}}N)$ sequential unentangling	$O(n^{3k-2} + kn^k N)$ directional gradient

Landau et al. arXiv:2410.23618, Leone et al. Quantum 2024, Cramer et al. Nat Comm 2010, Arunachalam et al. TQC 2023

Work vs complexity



For these special classes of states, we give provably-efficient energy-optimal erasure protocols based on *learning*.

What about more general states?

Can we achieve Landauer's limit in polynomial time?

Computational hardness

Can we achieve Landauer's limit in polynomial time for general states?

No!

There is a class of states for which the Landauer's limit is

$$\Theta(n\,\mathrm{polylog}(n))kT\ln 2,$$

independent of N

but any polynomial time quantum algorithm must pay

$$W_{\mathrm{Haar}} = \log_2 \binom{N+2^n-1}{N} kT \ln 2$$

#qubits

$$\sim nN(1-o(1))$$

omitting $1/\mathrm{poly}(n), \log(1/\epsilon)$

uncertainty principle

failure probability

$N = \mathrm{poly}(n)$

joules of work to erase them!

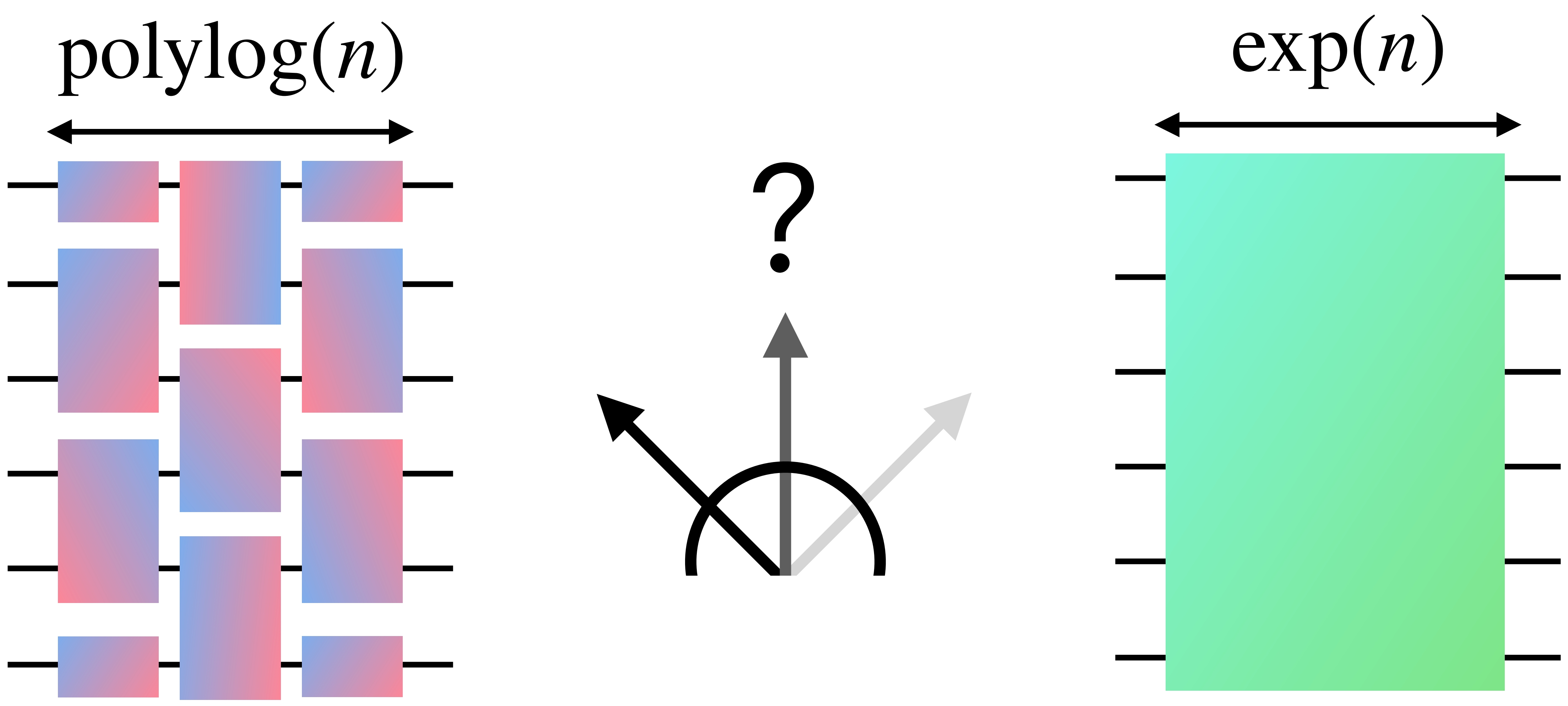
Pseudorandom states

Under standard cryptographic assumption,

existence of one-way functions secure against any sub-exponential time quantum adversary

pseudorandom states can be constructed in $d = \mathrm{polylog}(n)$ depth.

They cannot be efficiently distinguished from Haar random states with non-negligible probability.



Ji et al. CRYPTO 2018, Schuster et al. arXiv:2407.07754

Pseudorandom states

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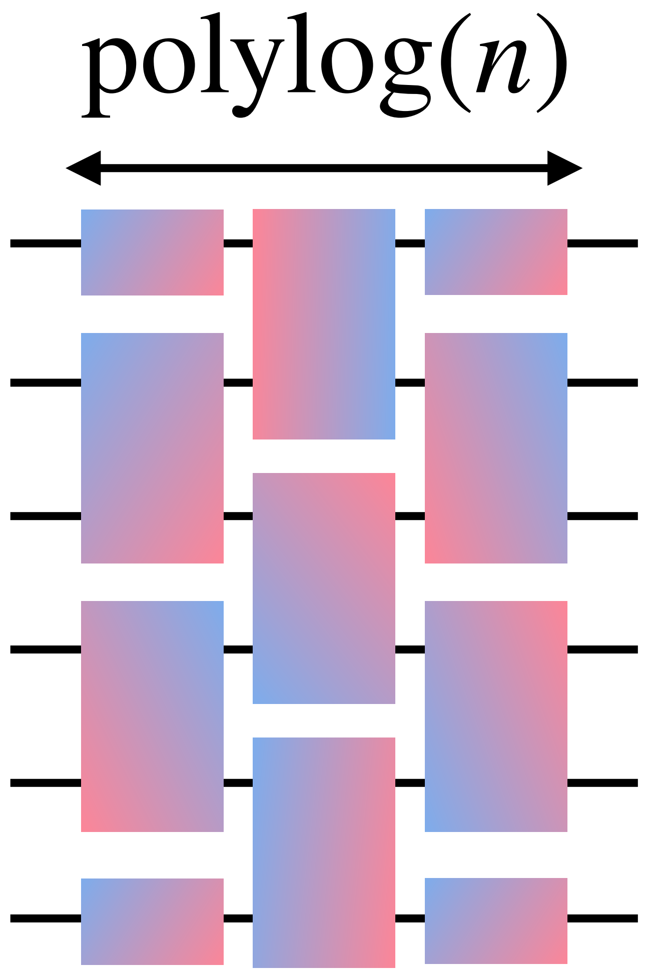
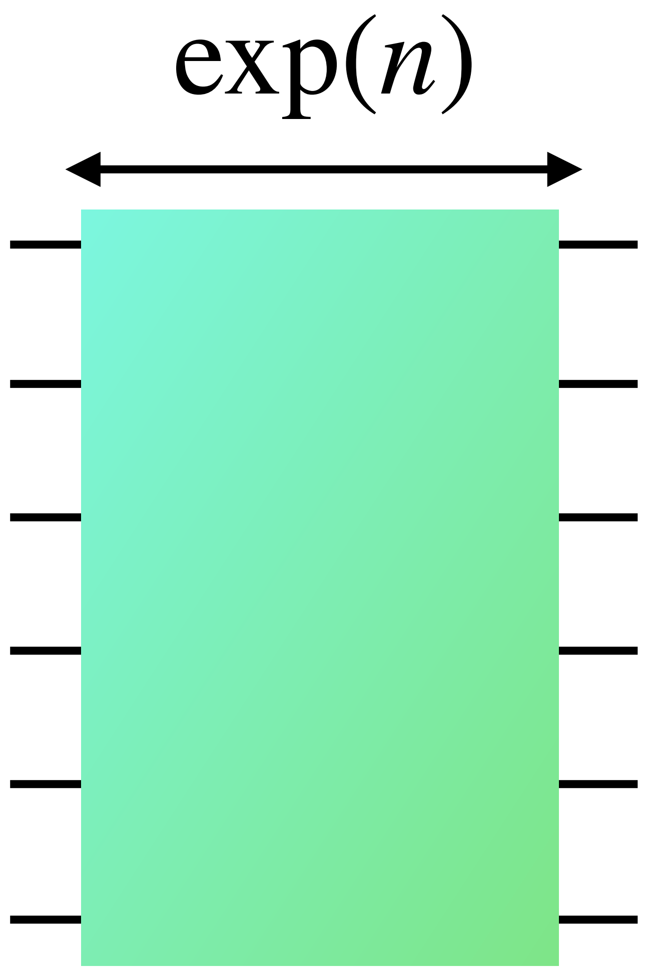


$$W = \Theta(n\mathrm{polylog}(n))kT\ln 2$$



$$W_{\mathrm{Haar}} = \log_2 \binom{N + 2^n - 1}{N} kT \ln 2$$

Full reduction:

1.erase

2.test if erase succeeded

3.measure work cost

Measuring the work cost of erasure gives a way to distinguish them!

=> no polynomial time quantum algorithm can achieve Landauer's limit!

a much stronger no-go result compared to the 3rd law of thermodynamics

(this is a genuine quantum many-body phenomenon!)

Summary

- A rigorous connection between thermodynamics and quantum learning theory.
- This allows us to answer several fundamental questions:

Learning:

Learning has no fundamental energy cost itself.

Our (in)ability to learn significantly impact the energy cost of thermodynamic tasks.

omitted: energy gain in work extraction

Thermodynamics:

Learning provides provably-efficient energy-optimal protocols.

The complexity of quantum many-body systems leads to drastic change of fundamental physical laws.

e.g., Landauer's principle

Open questions

- Extension to more realistic scenarios & continuous variable systems
- Other thermodynamic tasks; other physical properties of learning itself
- Consequences in high energy physics (pseudorandom models of black hole)