

# Exponential Quantum Advantage in Processing Massive Classical Data

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with Alexander Zlokapa, Hartmut Neven, Ryan Babbush,  
John Preskill, Jarrod McClean, Hsin-Yuan Huang

[arXiv:2604.07639](https://arxiv.org/abs/2604.07639)



Caltech



Google  
Quantum AI

# Will QC impact our daily life? How?

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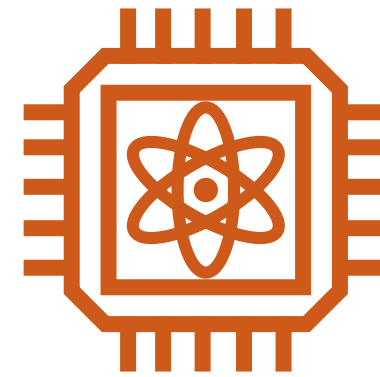


Google  
Quantum AI

# Utility of QC

**Specialized devices  
for niche tasks**

**General utility  
with wide impact**



# Utility of QC

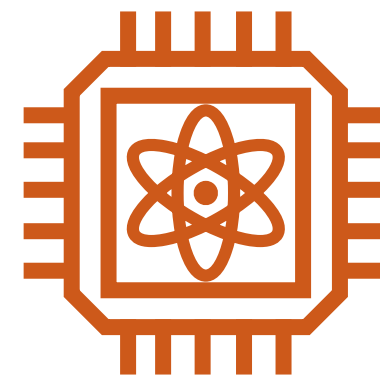
**Specialized devices  
for niche tasks**

**Cryptanalysis**

Shor'94

**Quantum Simulation**

Feynman'82, Lloyd'96



**General utility  
with wide impact**



# Utility of QC

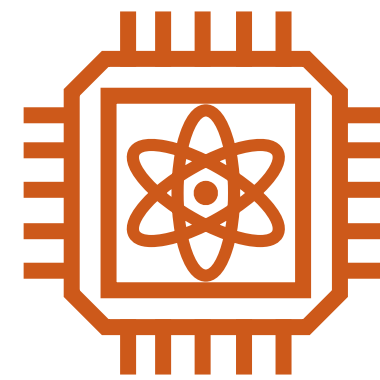
**Specialized devices  
for niche tasks**

Special structures  
exploitable by QC  
but classically hard

e.g., inherently quantum

Quantum Chemistry

Condensed Matter Physics



**General utility  
with wide impact**



# Utility of QC

**Specialized devices  
for niche tasks**

**Special structures  
exploitable by QC  
but classically hard**

e.g., inherently quantum

**Quantum Chemistry**

**Condensed Matter Physics**

**General utility  
with wide impact**

AI

Healthcare

Finance

**Unstructured  
massive classical data**

Media

Science



Engineering

# Life in a classical world



# Life in a classical world



## Reviews

“One of the best movies.”



“Fantastic production.”



“Story makes no sense.”



# Life in a classical world



## Reviews

“One of the best movies.” 




“Fantastic production.” 



“Story makes no sense.” 

## Classical Data

$\vec{x}_1$  

$\vec{x}_2$  

$\vec{x}_3$  


# Life in a classical world



Reviews

Classical Data

“One of the best movies.” 

$\vec{x}_1$  



“Fantastic production.” 

$\vec{x}_2$  



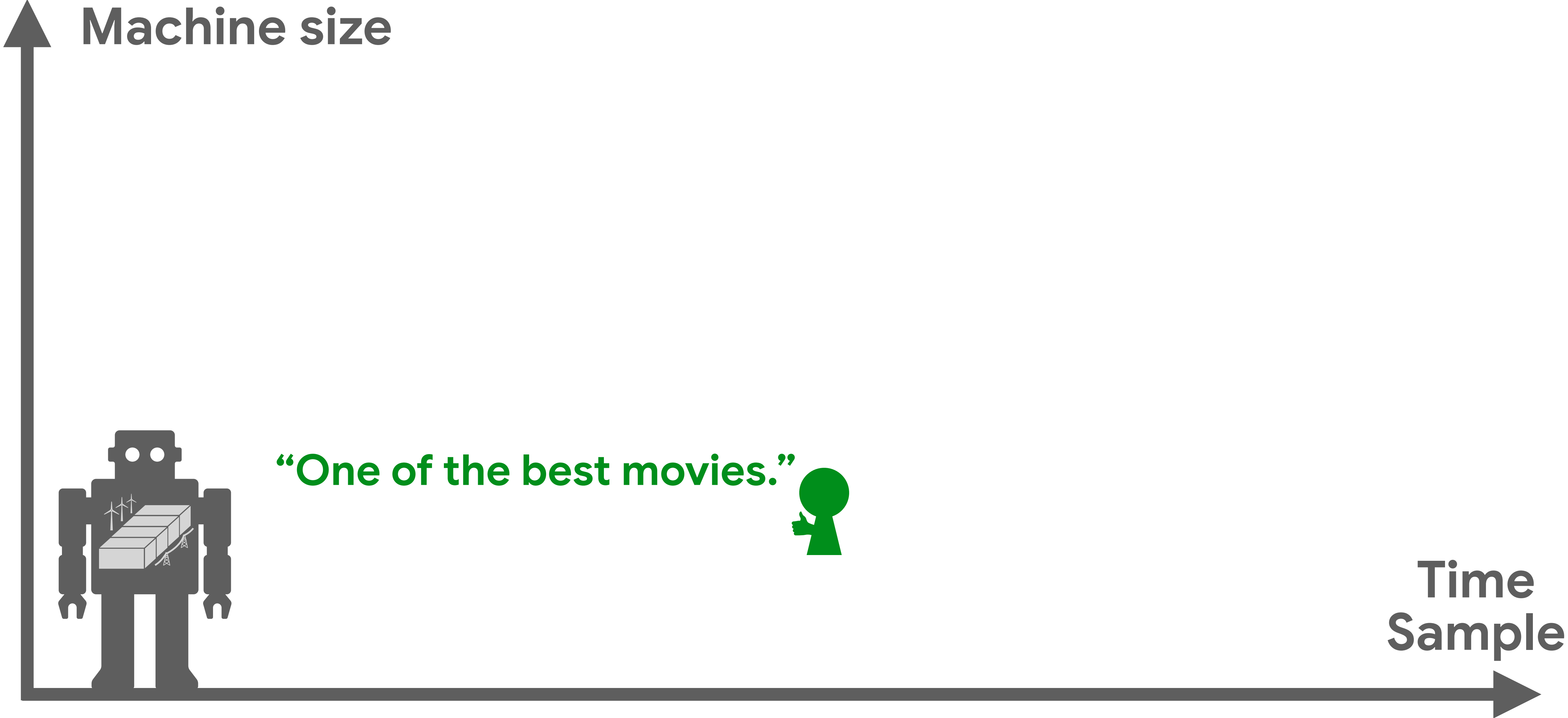
“Story makes no sense.” 

$\vec{x}_3$  

Task: sentiment analysis

$\vec{x}$  

# Classical ML



Machine size

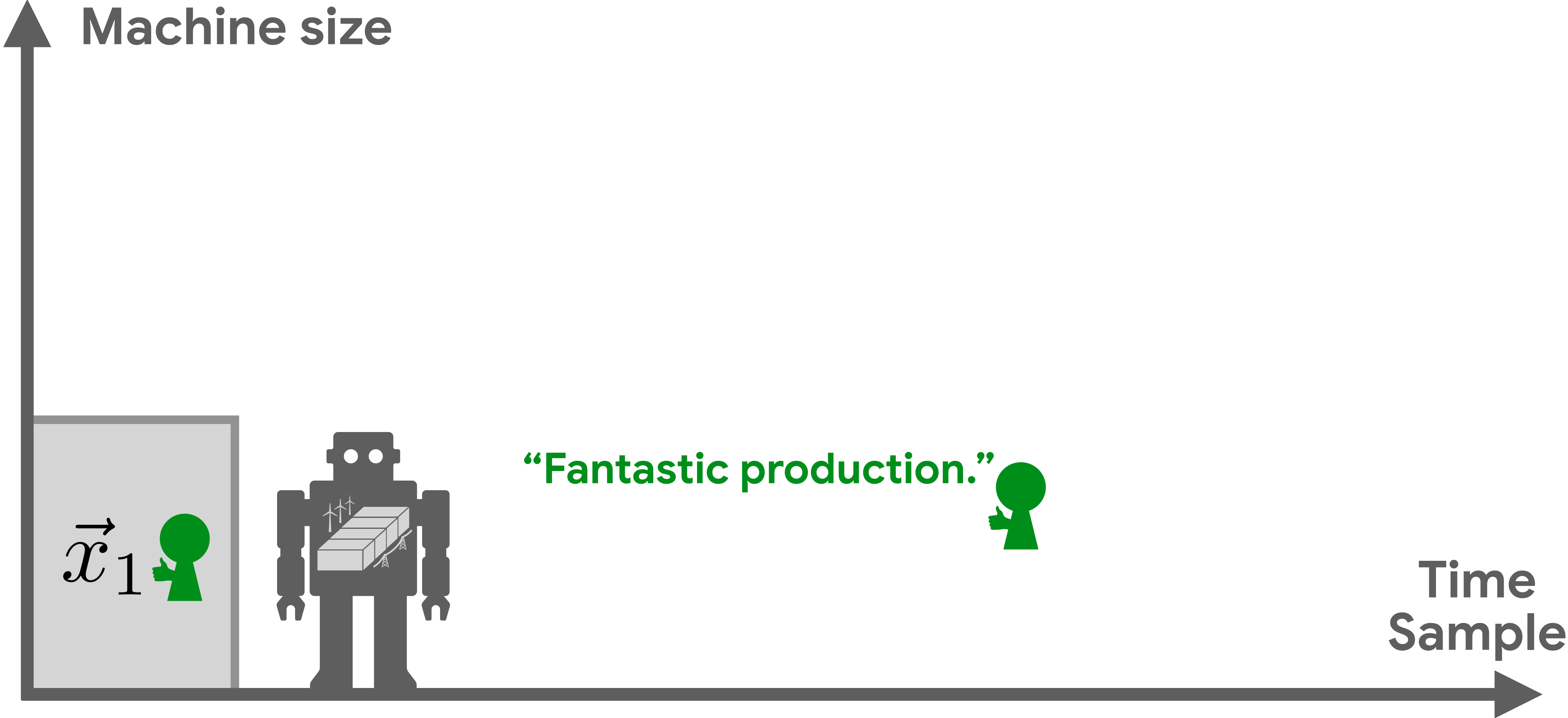
“One of the best movies.”

Time Sample

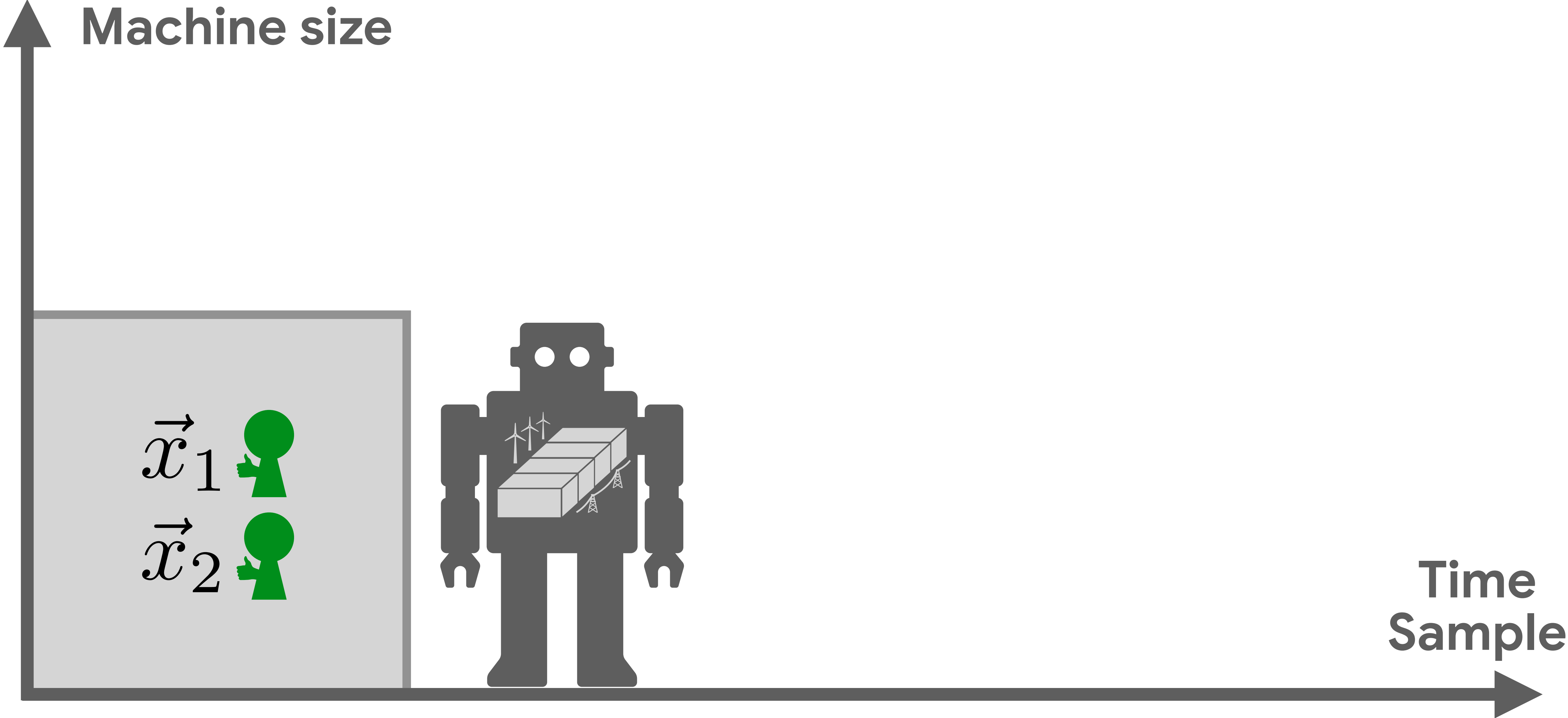
# Classical ML



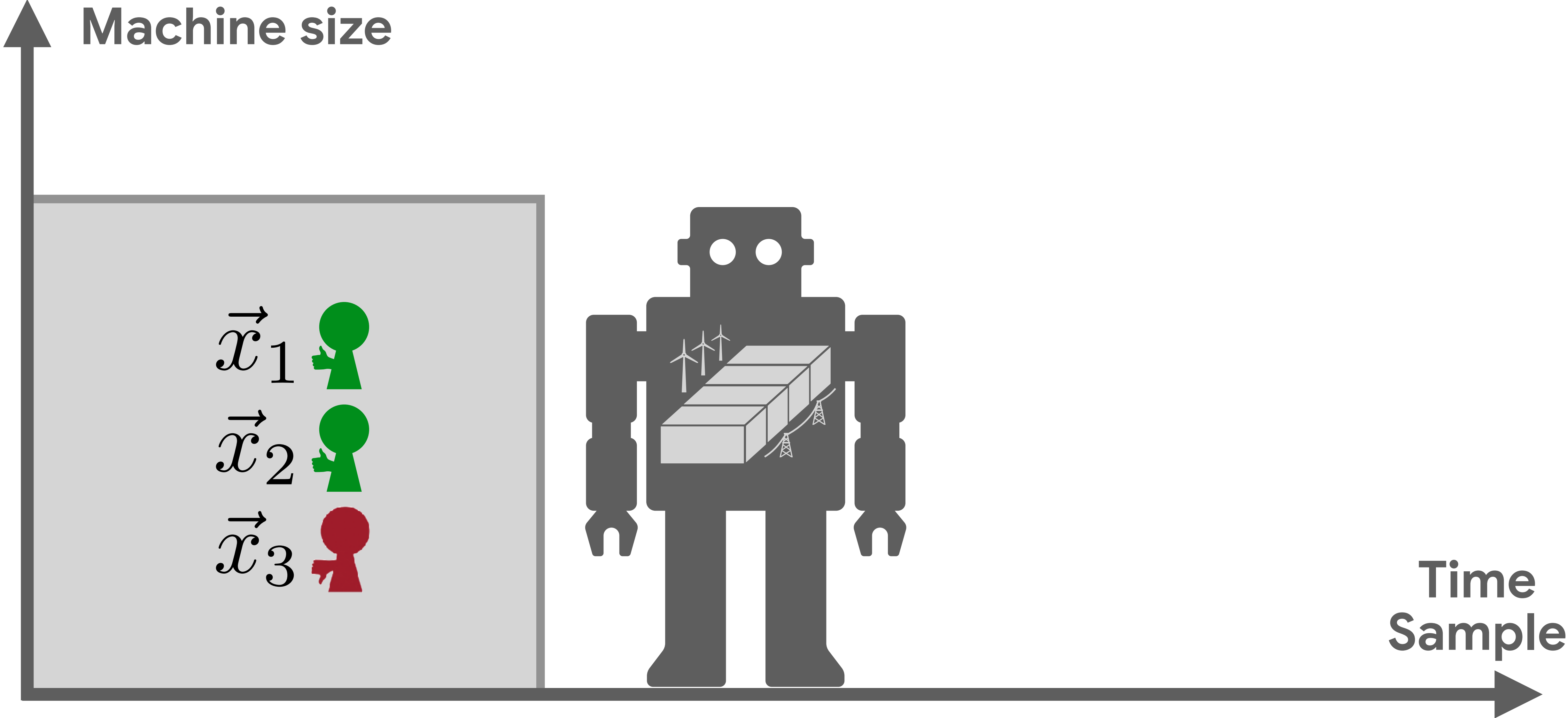
# Classical ML



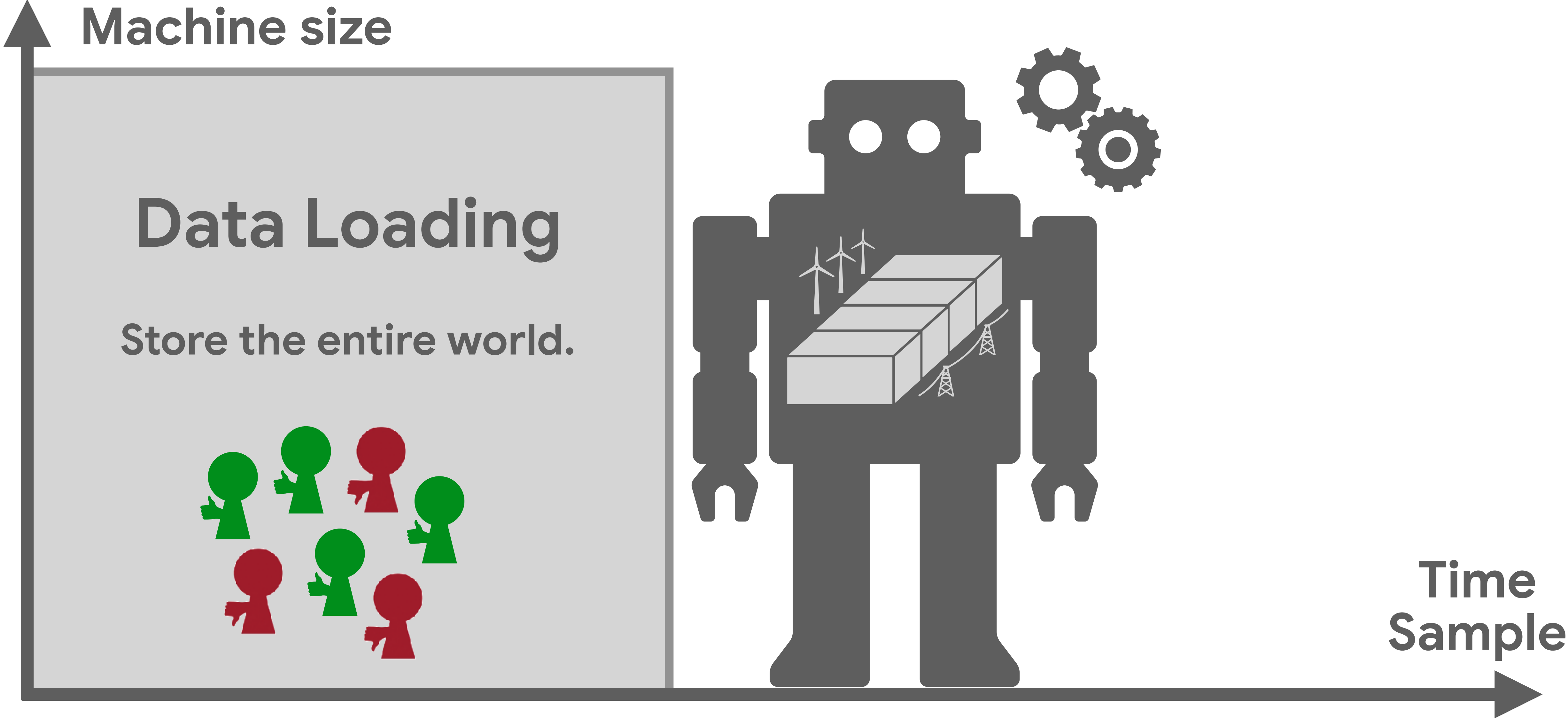
# Classical ML



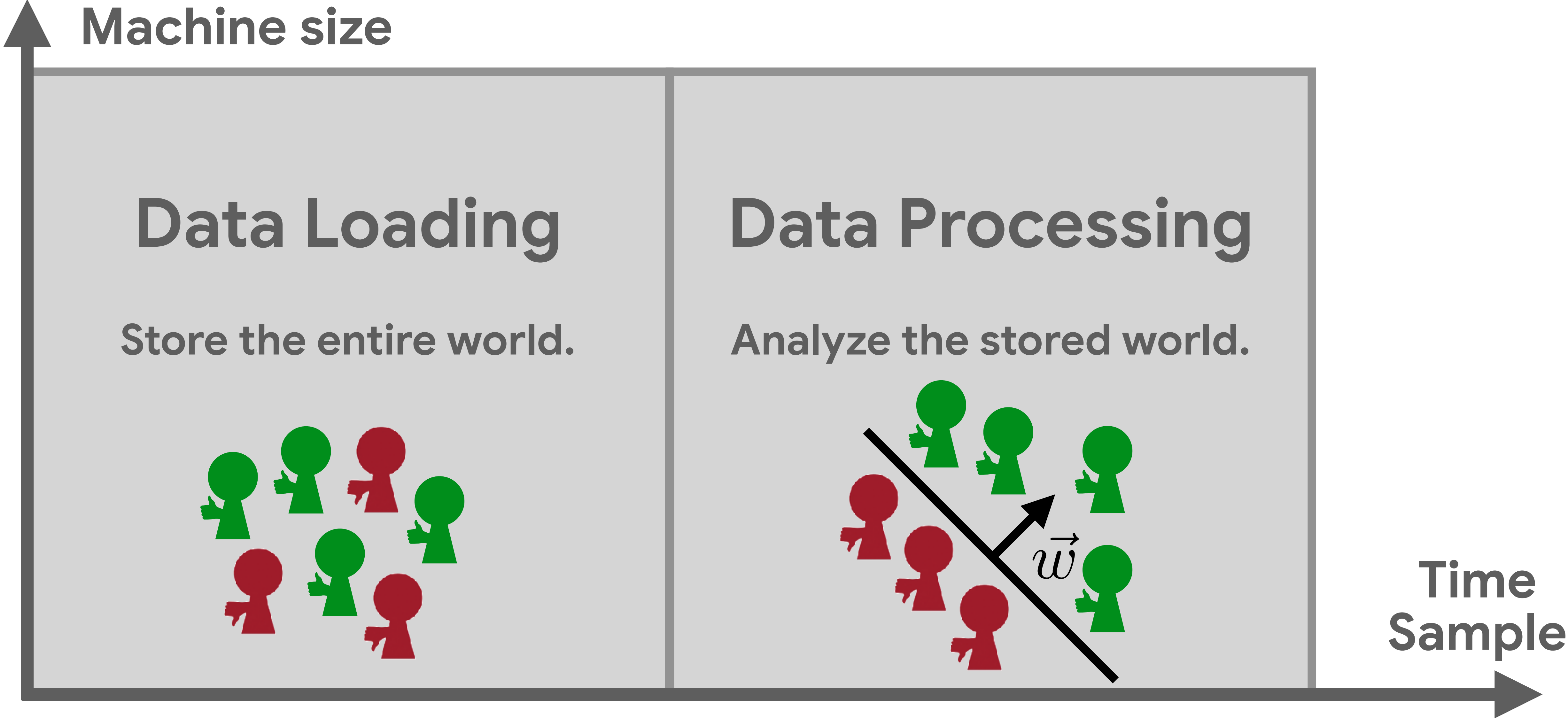
# Classical ML



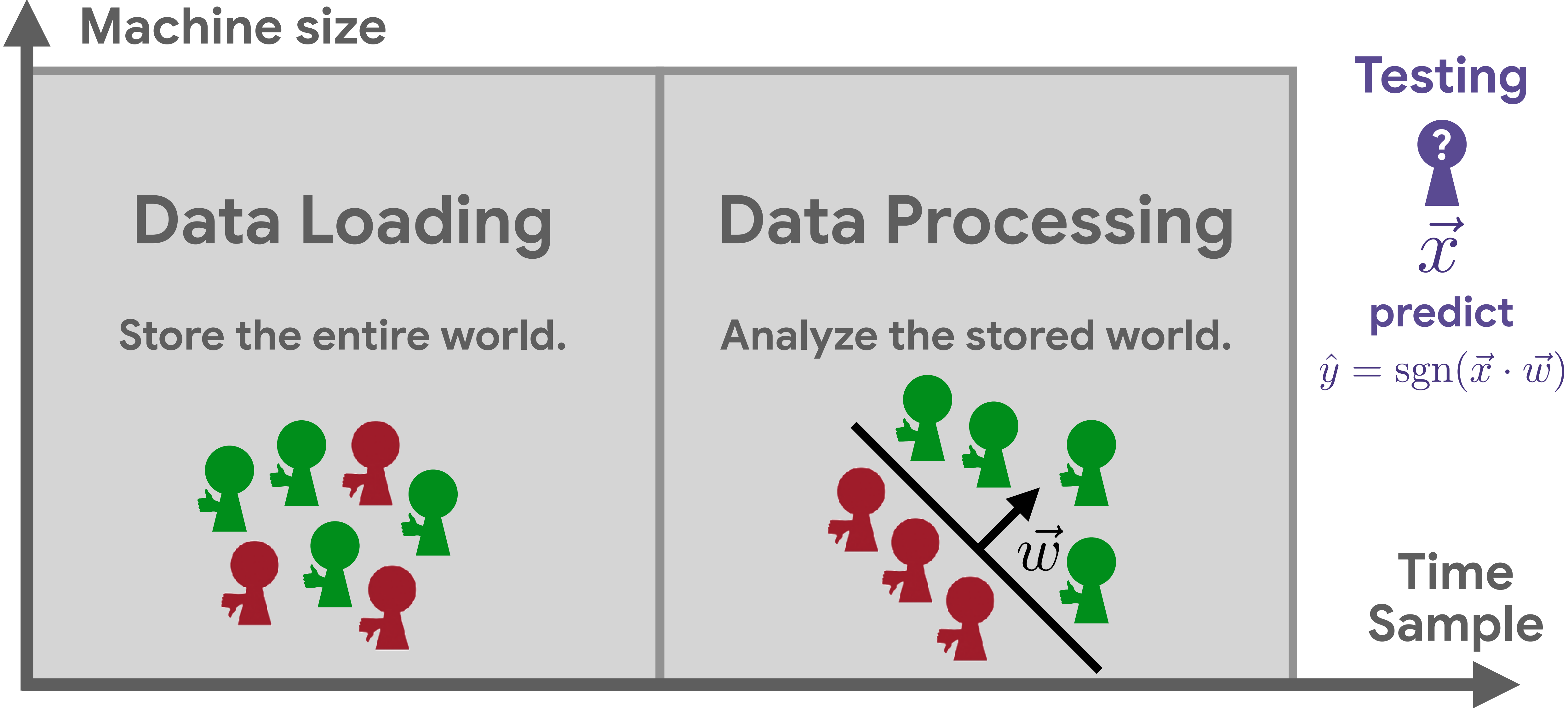
# Classical ML



# Classical ML



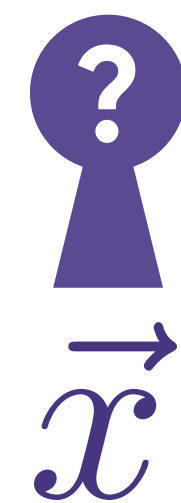
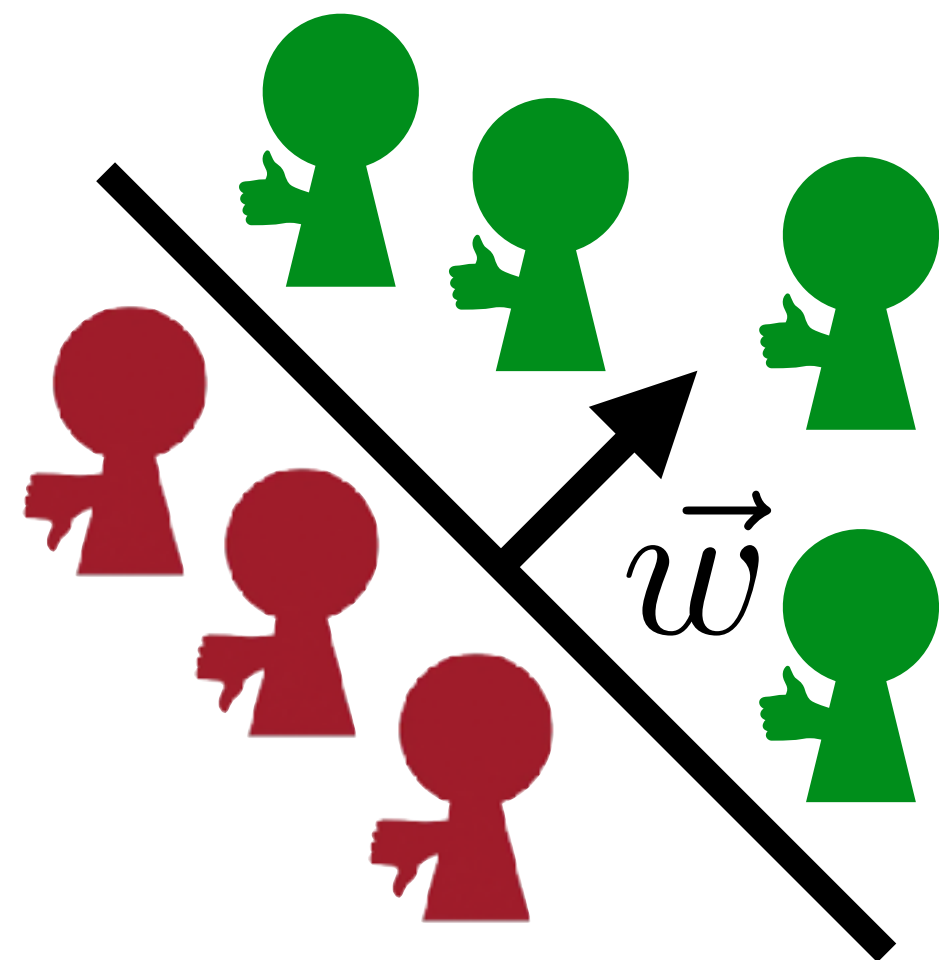
# Classical ML



# Main Results

**Classification:** given data  $(i, \vec{x}_i, y_i)$  randomly sampled from  $N$  examples, each with a  $D$ -dim sparse feature vector and a label, train a regularized  $D$ -dim classifier  $\vec{w}$  such that we can predict the label of any classifiable sparse test vectors.

margin not too small



**predict**

$$\hat{y} = \text{sgn}(\vec{x} \cdot \vec{w})$$

# Main Results

**Classification:** given data  $(i, \vec{x}_i, y_i)$  randomly sampled from  $N$  examples, each with a  $D$ -dim sparse feature vector and a label, train a regularized  $D$ -dim classifier  $\vec{w}$  such that we can predict the label of any classifiable sparse test vectors.

margin not too small

## Theorem 1 (Classification)

With  $\tilde{O}(N)$  samples, a quantum machine of  $\text{poly}(\log D)$  size can solve classification, whereas any classical machine of  $O(D^{0.99})$  size cannot.

# Main Results

**Classification:** often the training examples change over time, but the underlying classification rule remains the same.

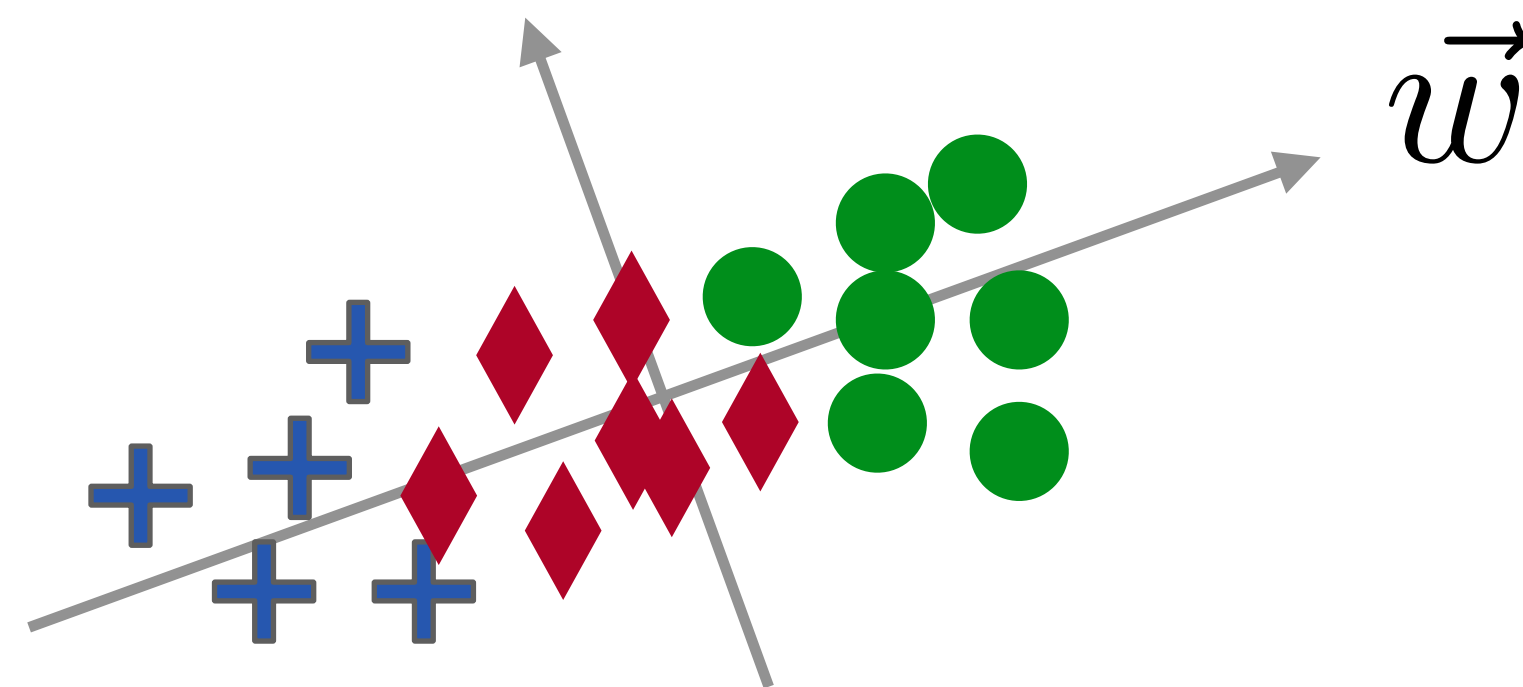
**Example:** large particle colliders

## Theorem 2 (Dynamic Classification)

A quantum machine of  $\text{poly}(\log D)$  size can solve classification using only  $\tilde{O}(N)$  samples, whereas any classical machine of  $O(D^{0.99})$  size requires at least  $\text{superpoly}(N)$  samples.

# Main Results

**Dimension reduction:** given data  $(i, \vec{x}_i)$  sampled from  $N$  examples, each with a  $D$ -dim sparse feature vector, learn an identifiable  $D$ -dim **principal component**  $\vec{w}$  such that we can eigenvalue gapped, non-negligible overlap with a prominent feature predict the 1D representation of any new sparse test vectors.



**unsupervised**

# Main Results

**Dimension reduction:** given data  $(i, \vec{x}_i)$  sampled from  $N$  examples, each with a  $D$ -dim sparse feature vector, learn an identifiable  $D$ -dim **principal component**  $\vec{w}$  such that we can eigenvalue gapped, non-negligible overlap with a prominent feature predict the 1D representation of any new sparse test vectors.

## Theorem 3 (Dim Reduction)

With  $\tilde{O}(N)$  samples, a **quantum machine of  $\text{poly}(\log D)$  size** can solve dim. reduct., whereas **any classical machine of  $O(D^{0.99})$  size** cannot.

# Main Results

**Dimension reduction:** given data  $(i, \vec{x}_i)$  sampled from  $N$  examples, each with a  $D$ -dim sparse feature vector, learn an identifiable  $D$ -dim **principal component**  $\vec{w}$  such that we can eigenvalue gapped, non-negligible overlap with a prominent feature predict the 1D representation of any new sparse test vectors.

## Theorem 4 (Dynamic Dim Reduction)

A **quantum machine of  $\text{poly}(\log D)$  size** can solve dim. reduct. using only  **$\tilde{O}(N)$  samples**, whereas **any classical machine of  $O(D^{0.99})$  size** requires at least **superpoly( $N$ ) samples**.

# Theory in Practice

IMDb

Movie review sentiment analysis

PBMC68k

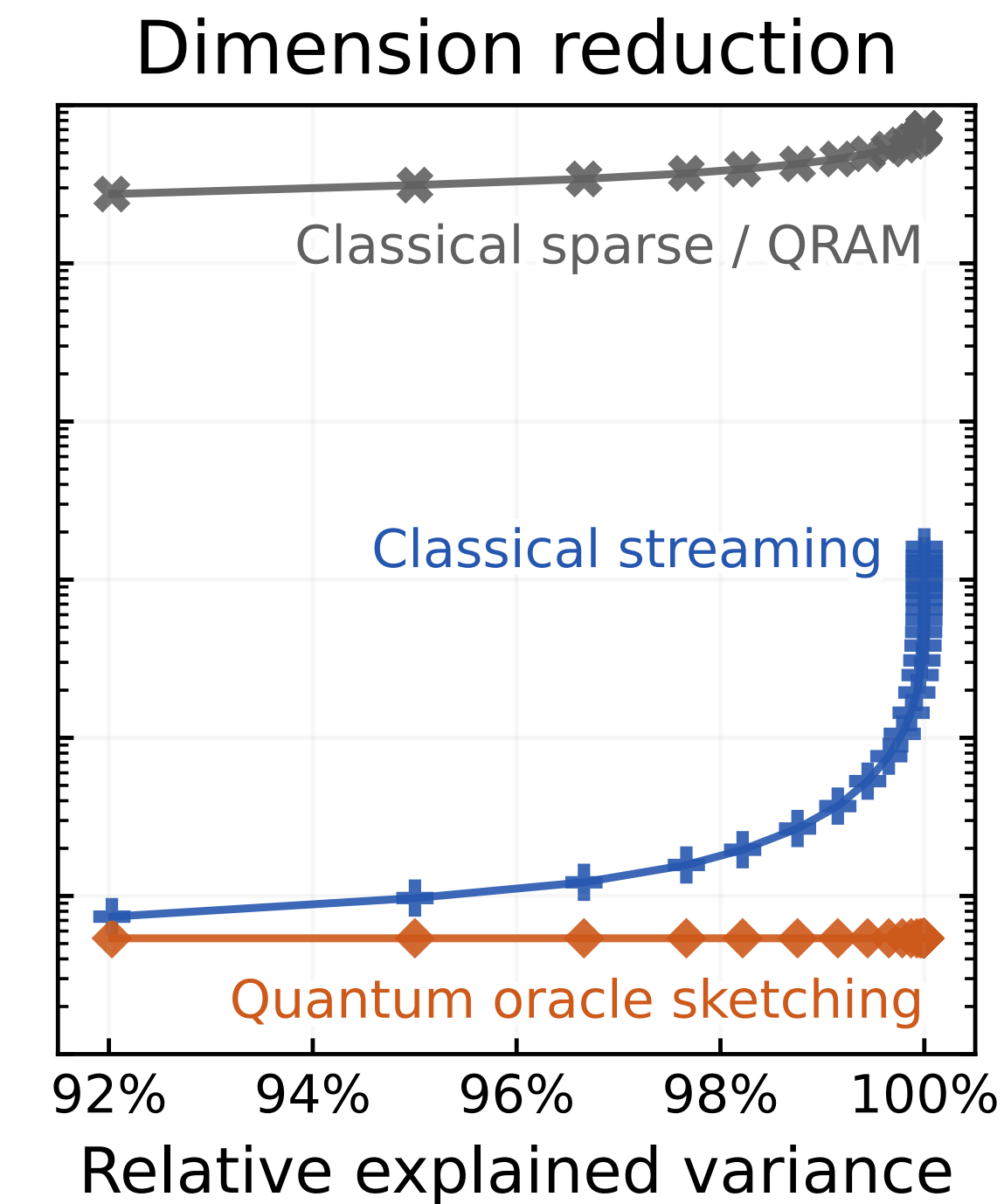
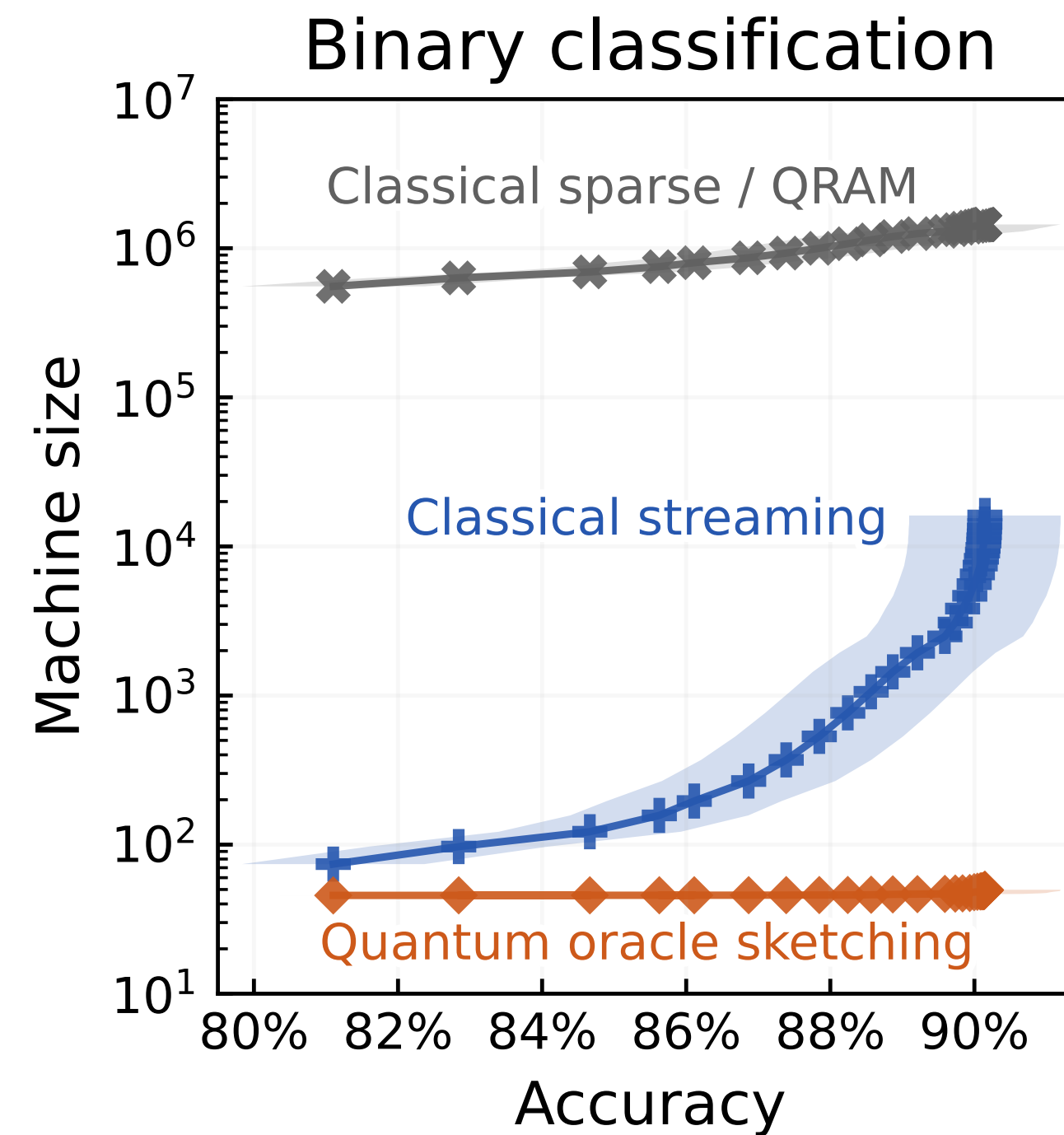
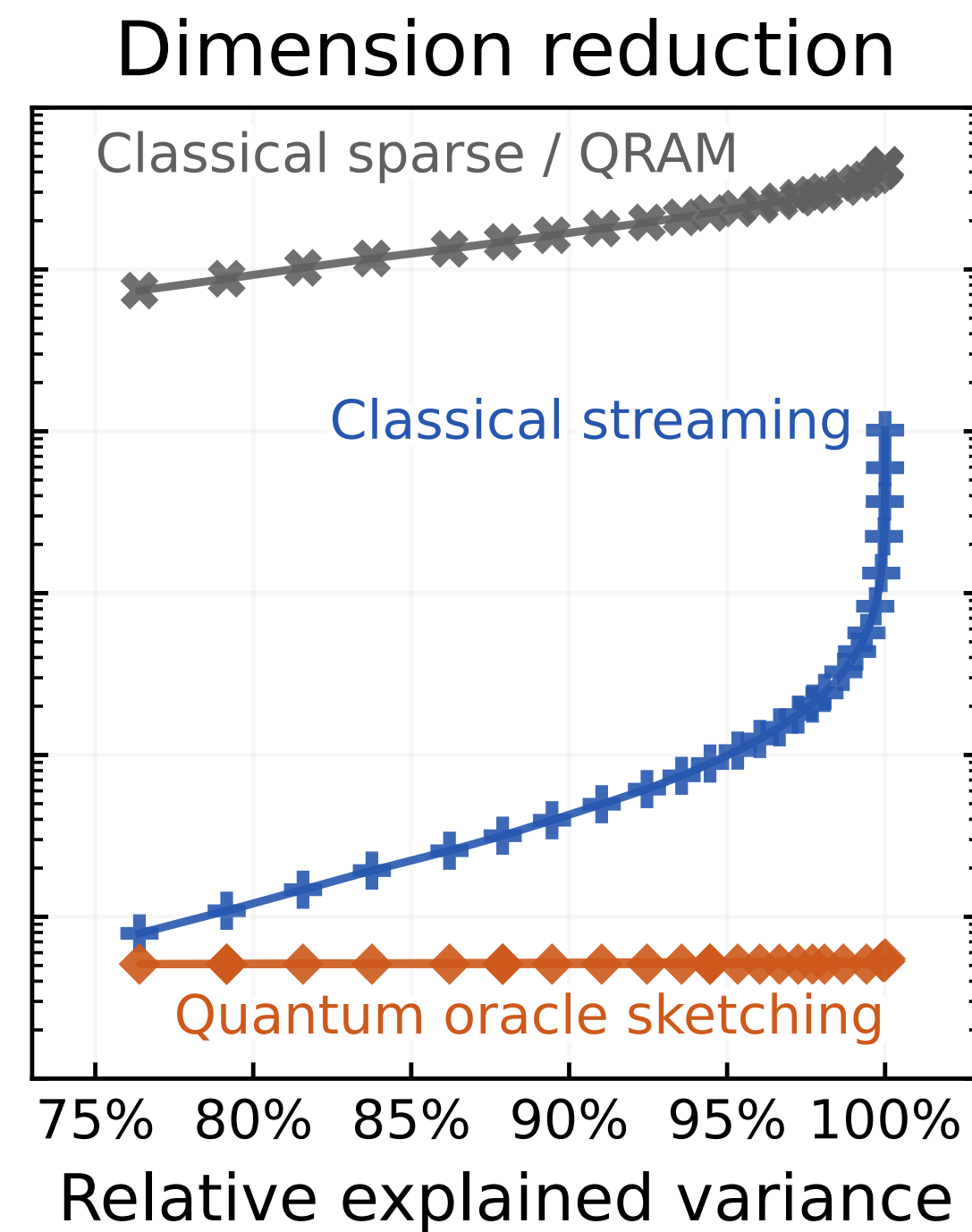
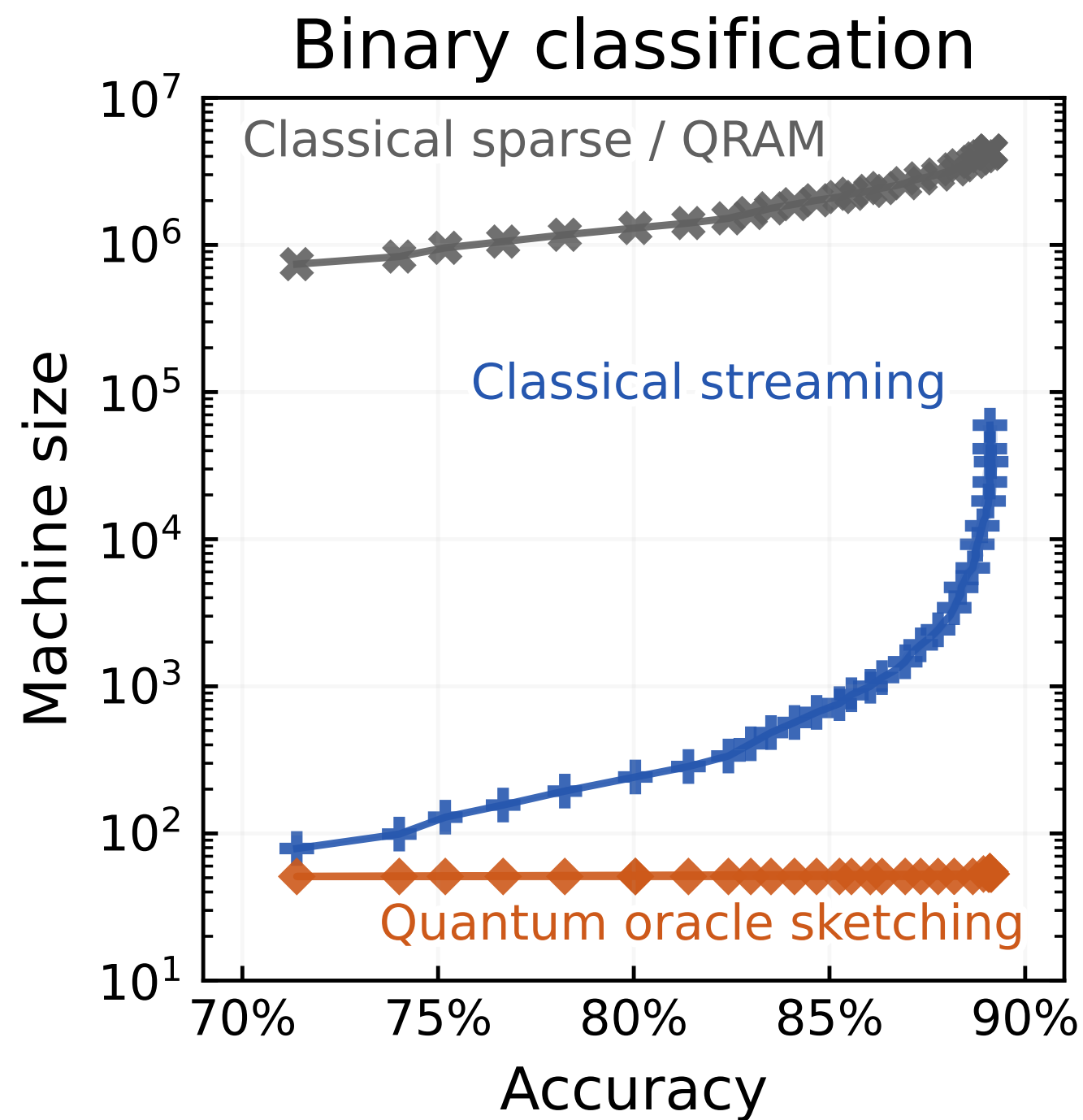
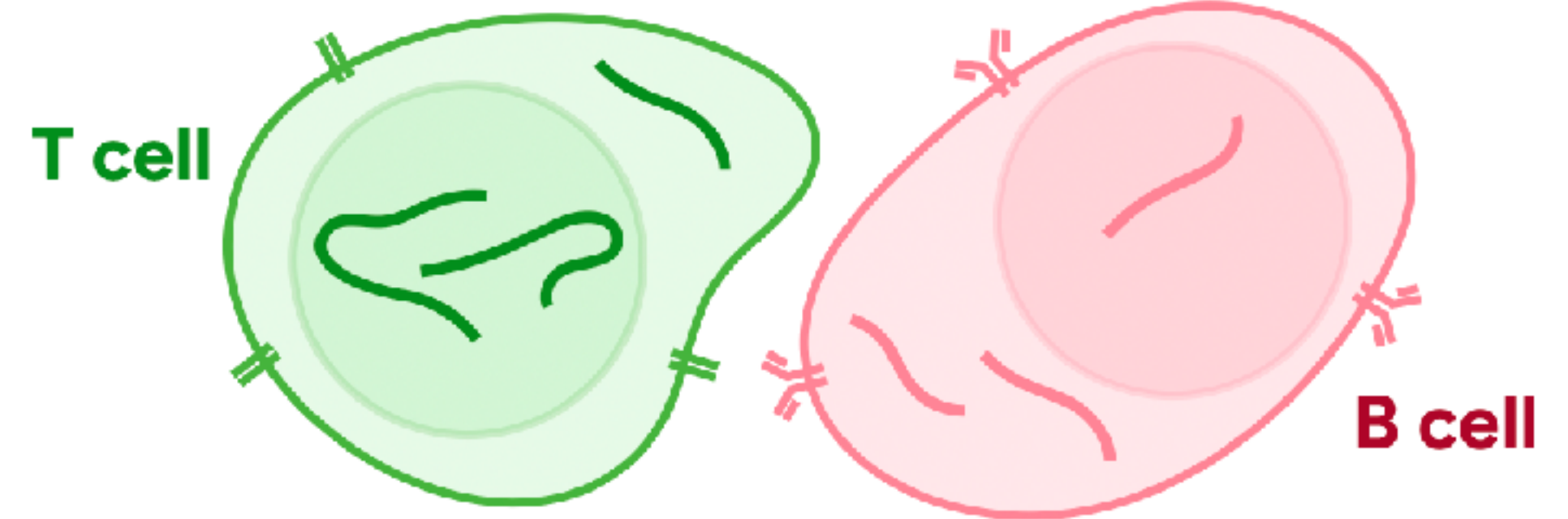
Single cell RNA sequencing



“One of the best movies.”



“Story makes no sense.”



# Theory in Practice

20Newsgroup

**Social media topic analysis**



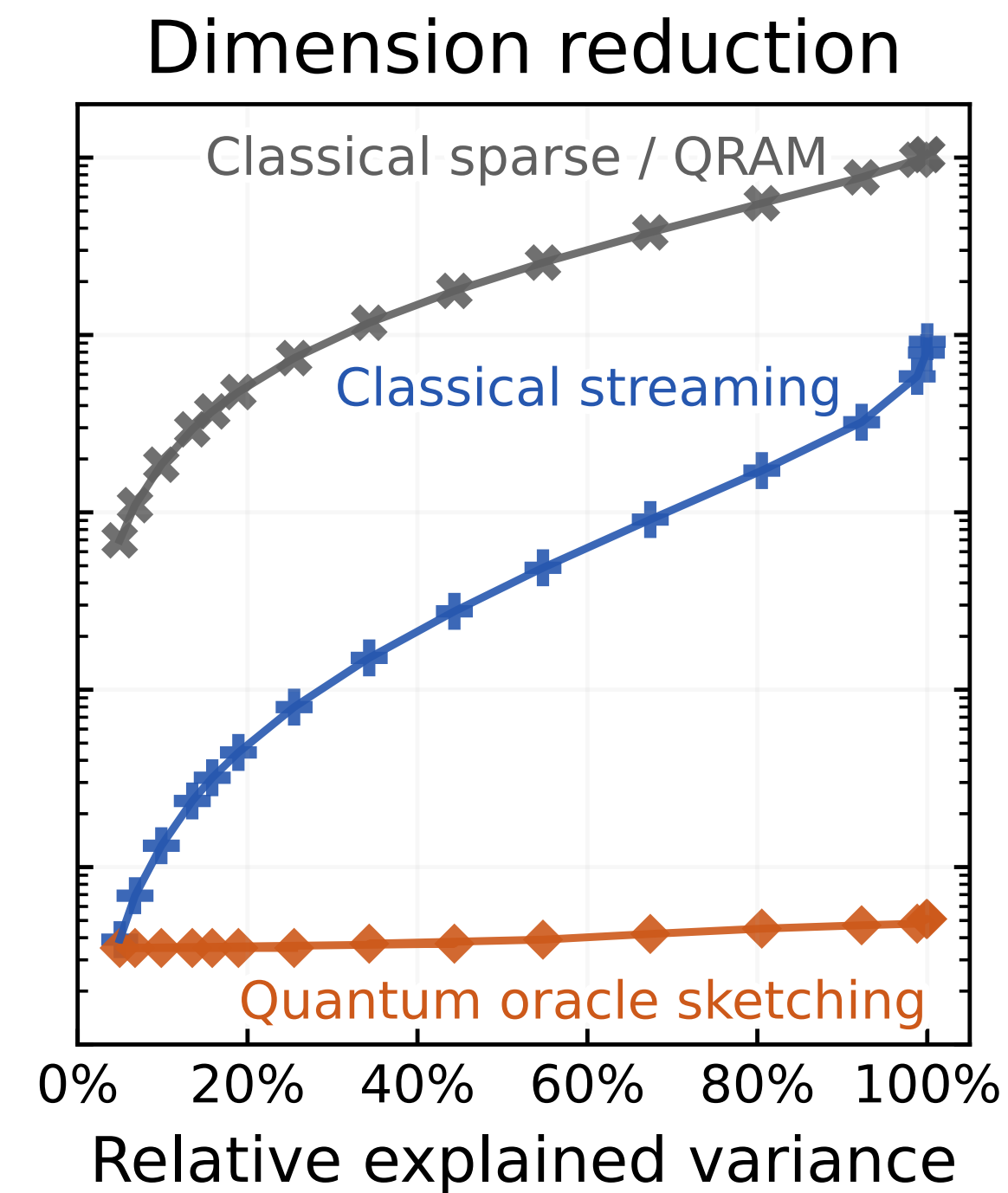
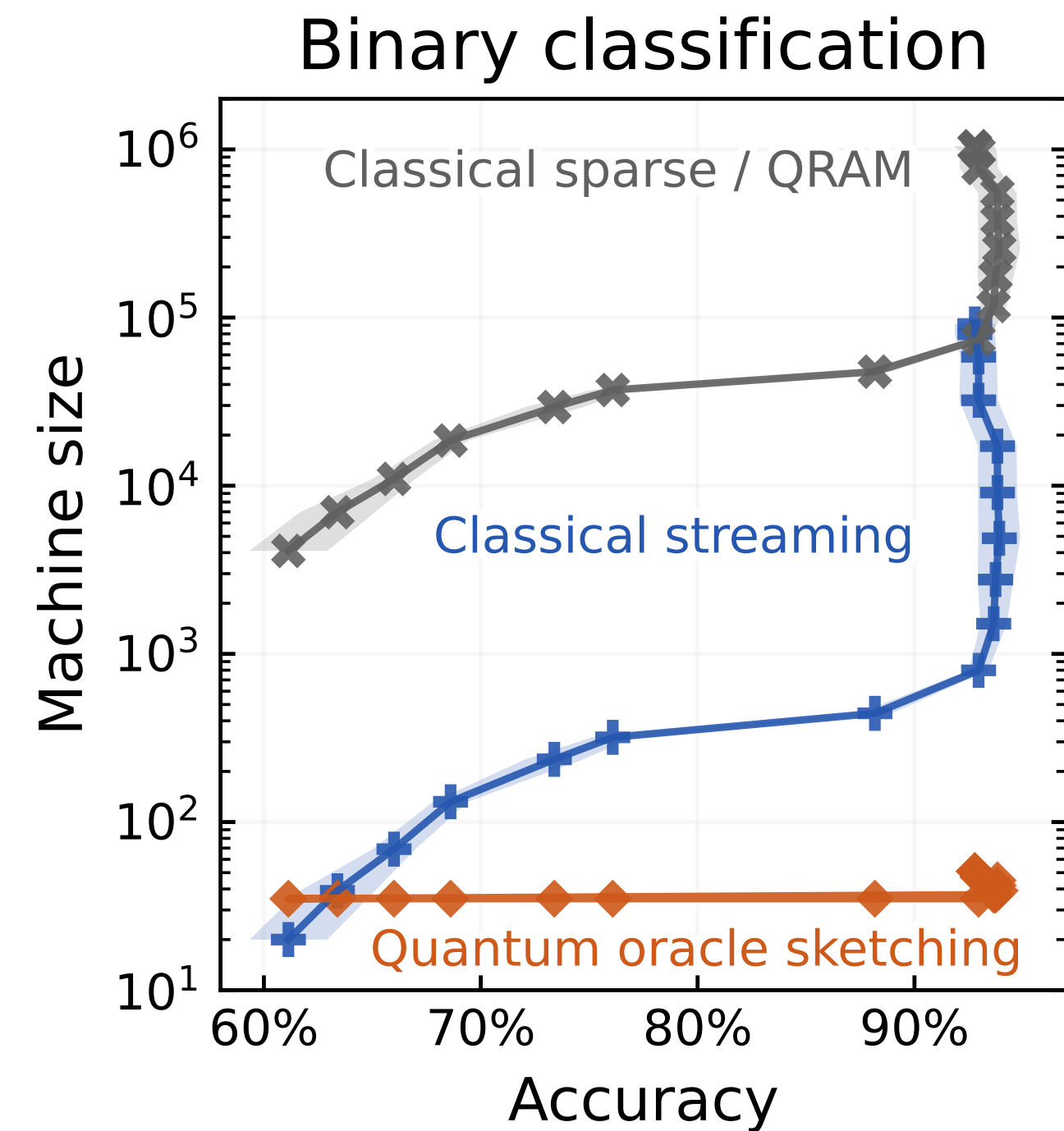
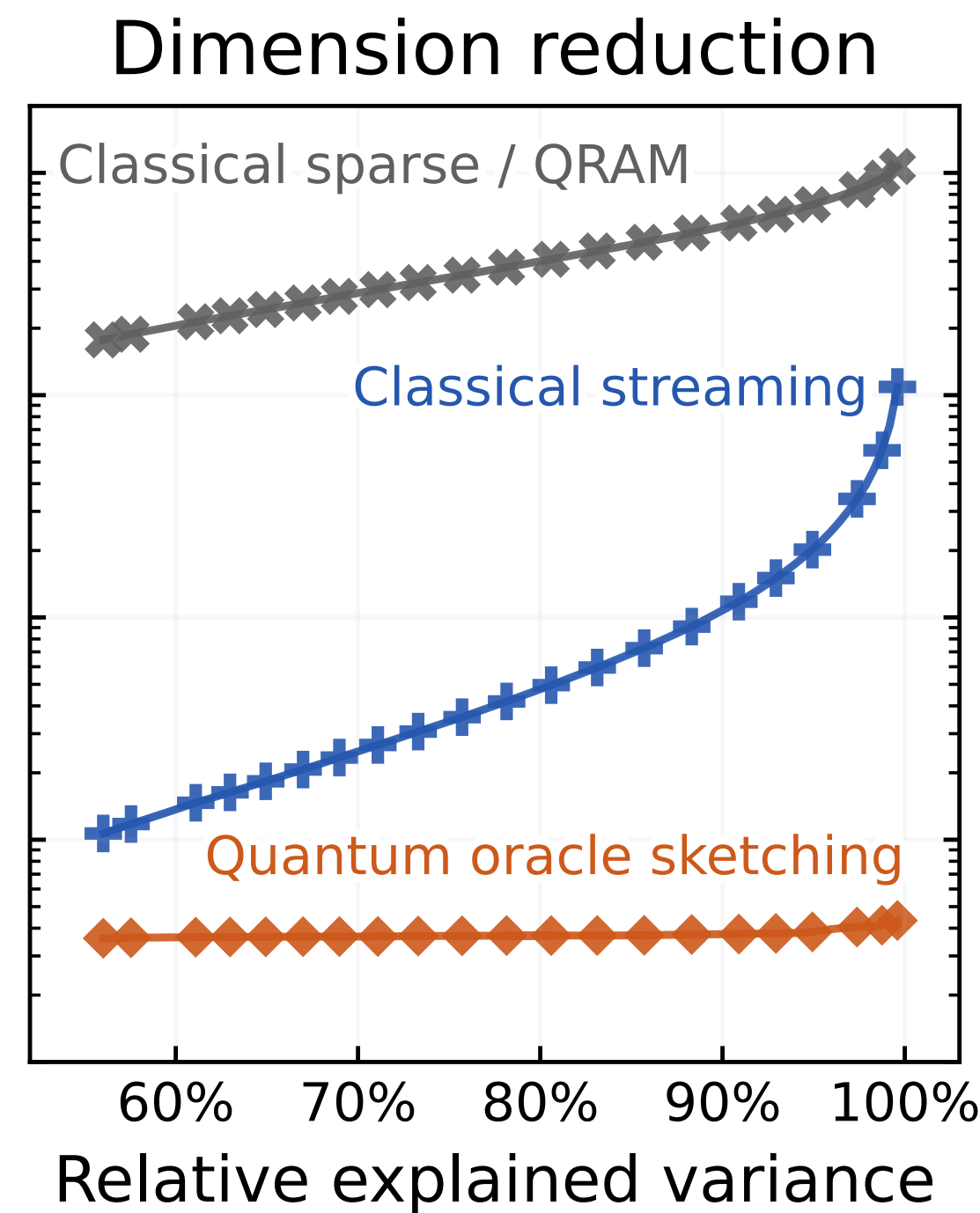
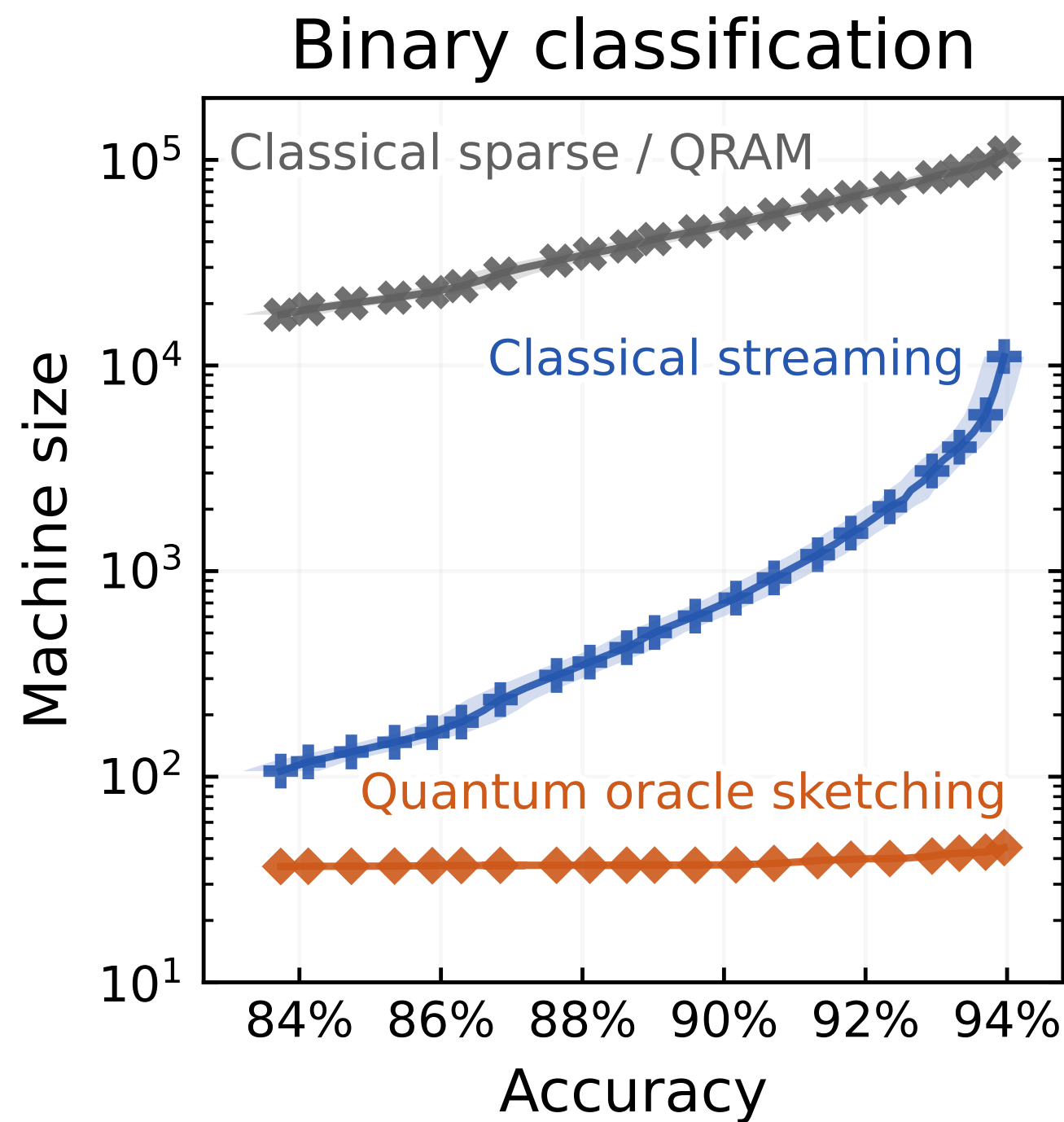
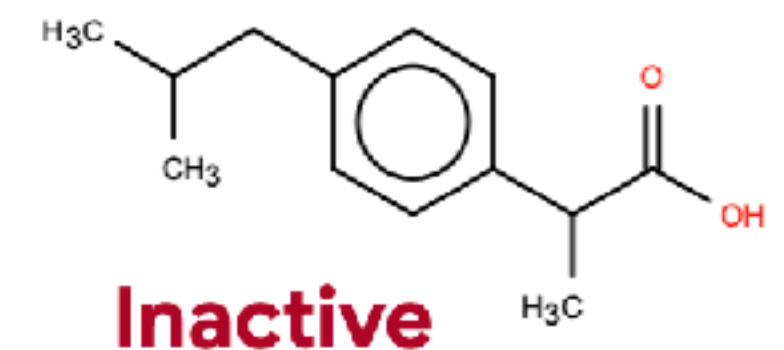
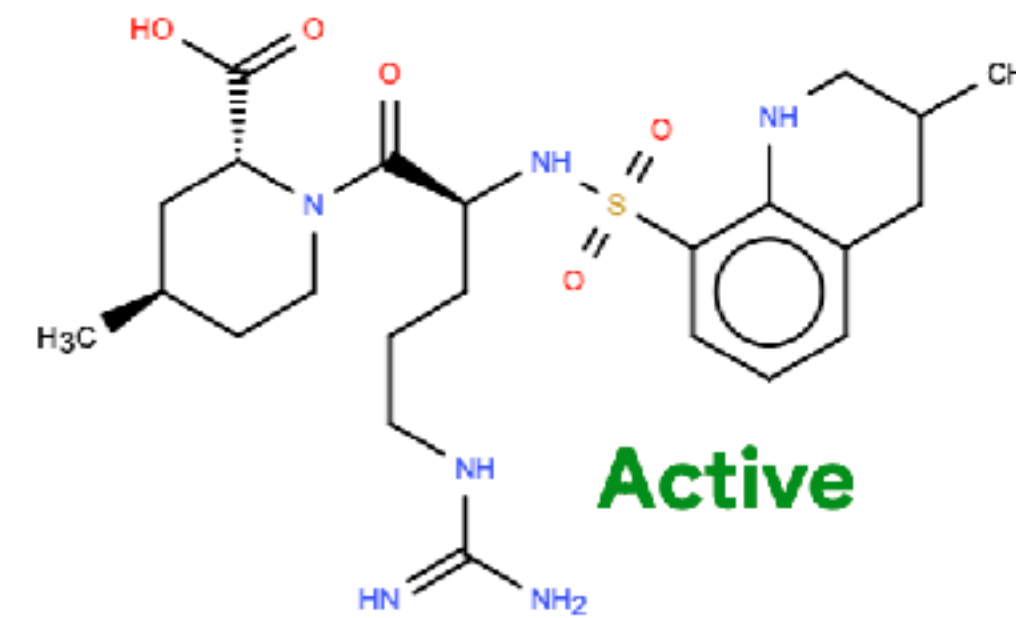
**Good offer at our local dealership.**



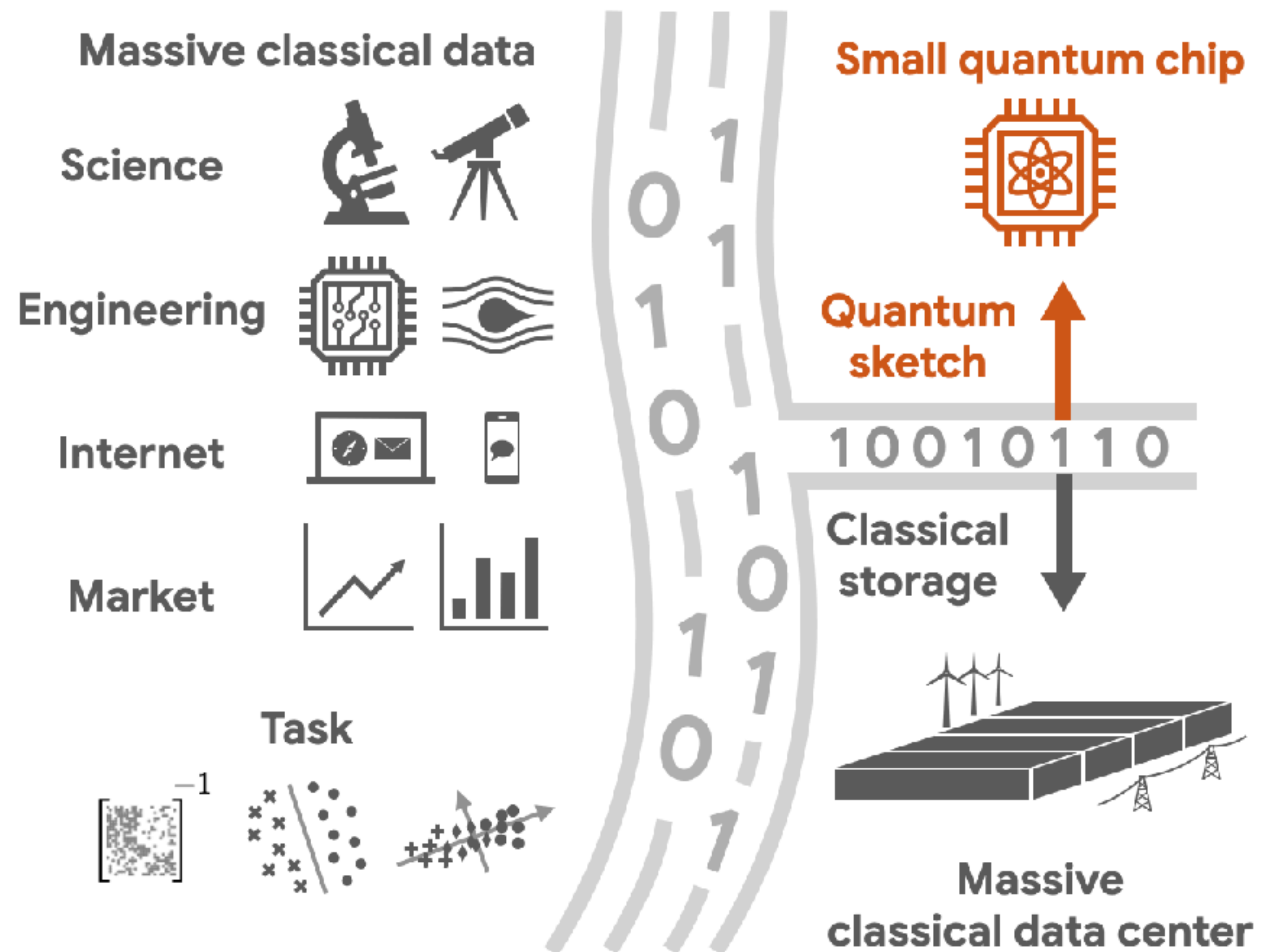
**Any opinions on this new medication?**

Thrombin

**Pharmaceutical drug discovery**

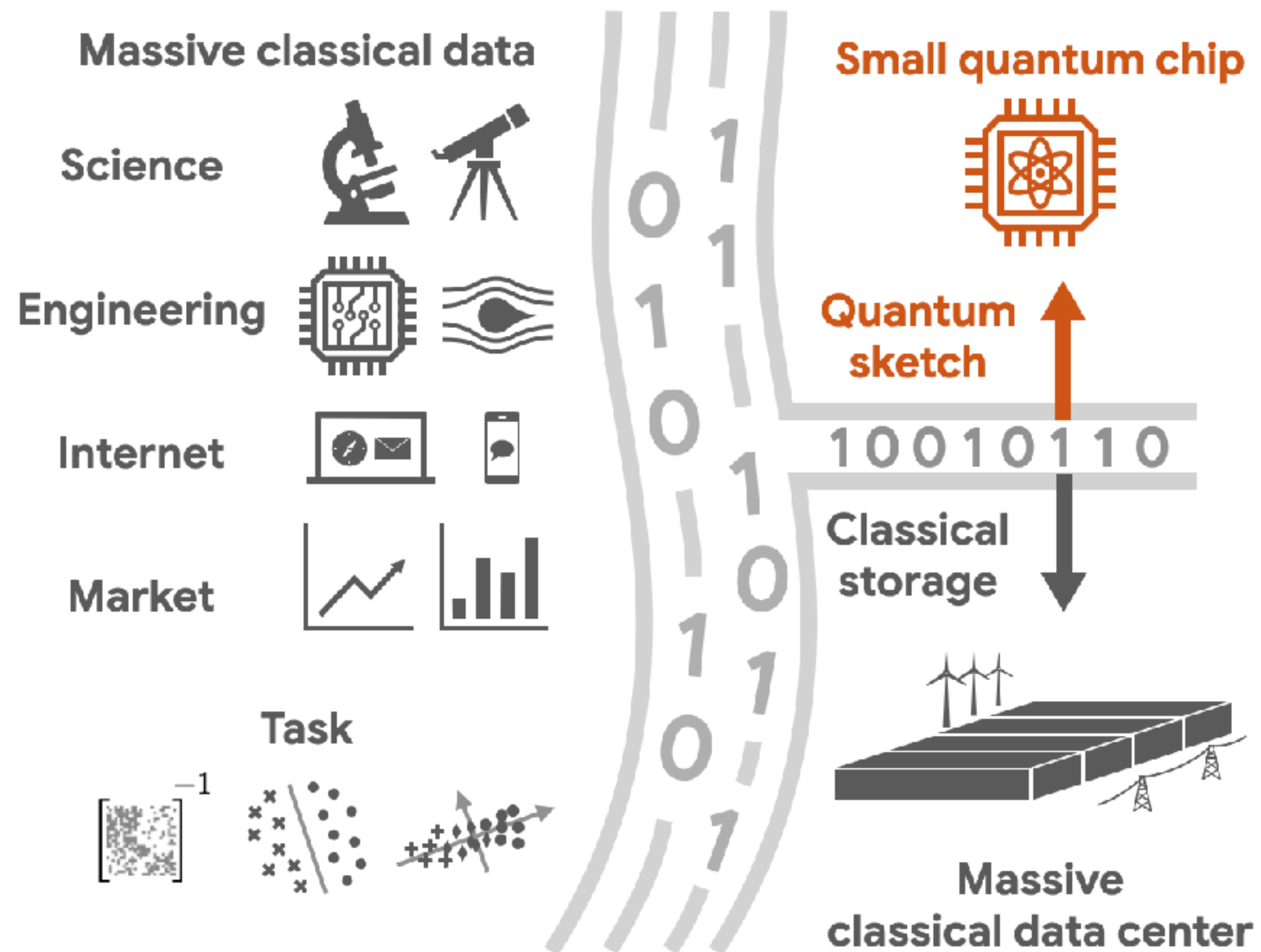


# Main Results



**Machine learning**  
is a broad domain of  
**exponential quantum**  
advantage.

# Main Results

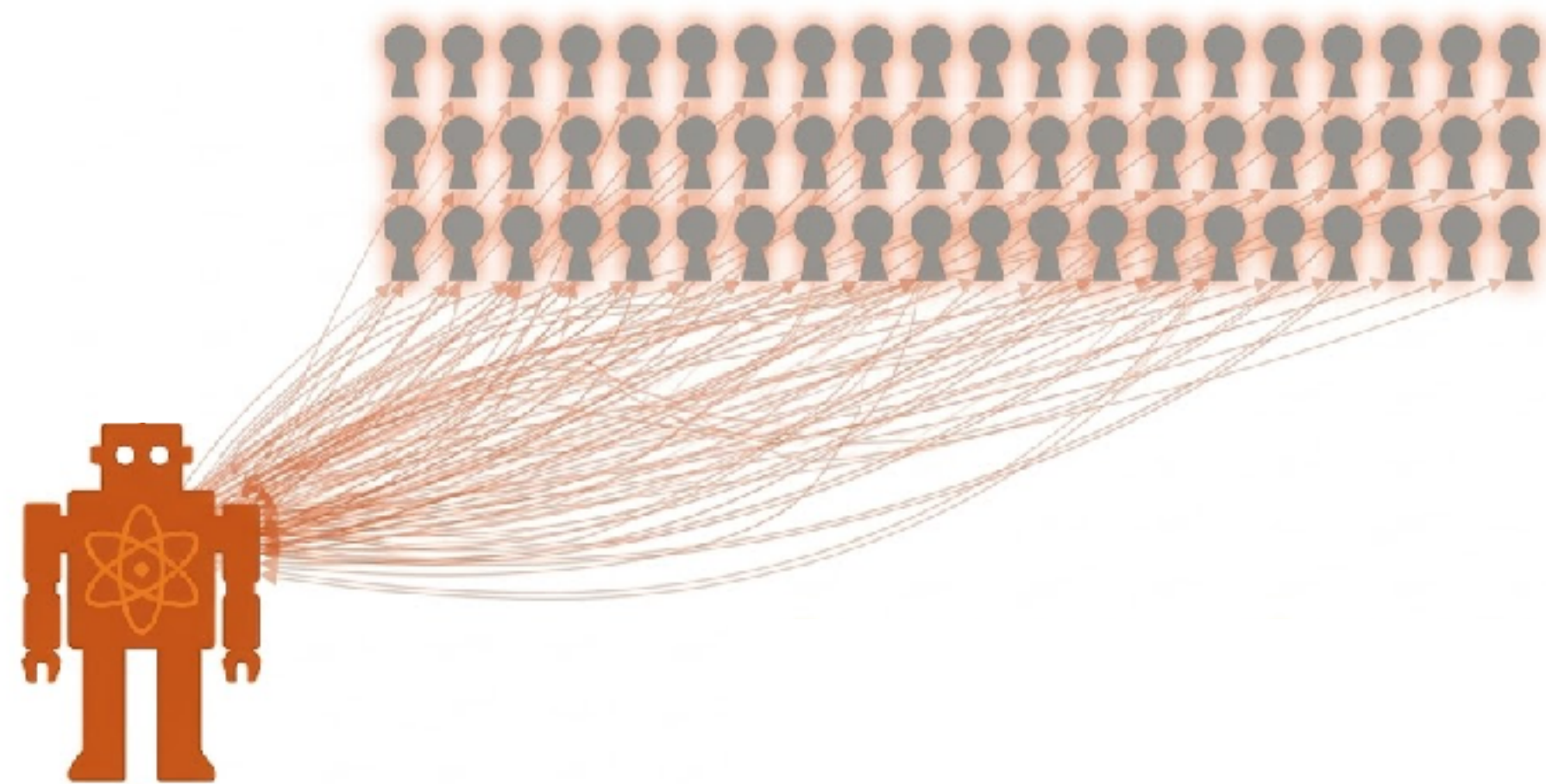


**Machine learning**  
is a broad domain of  
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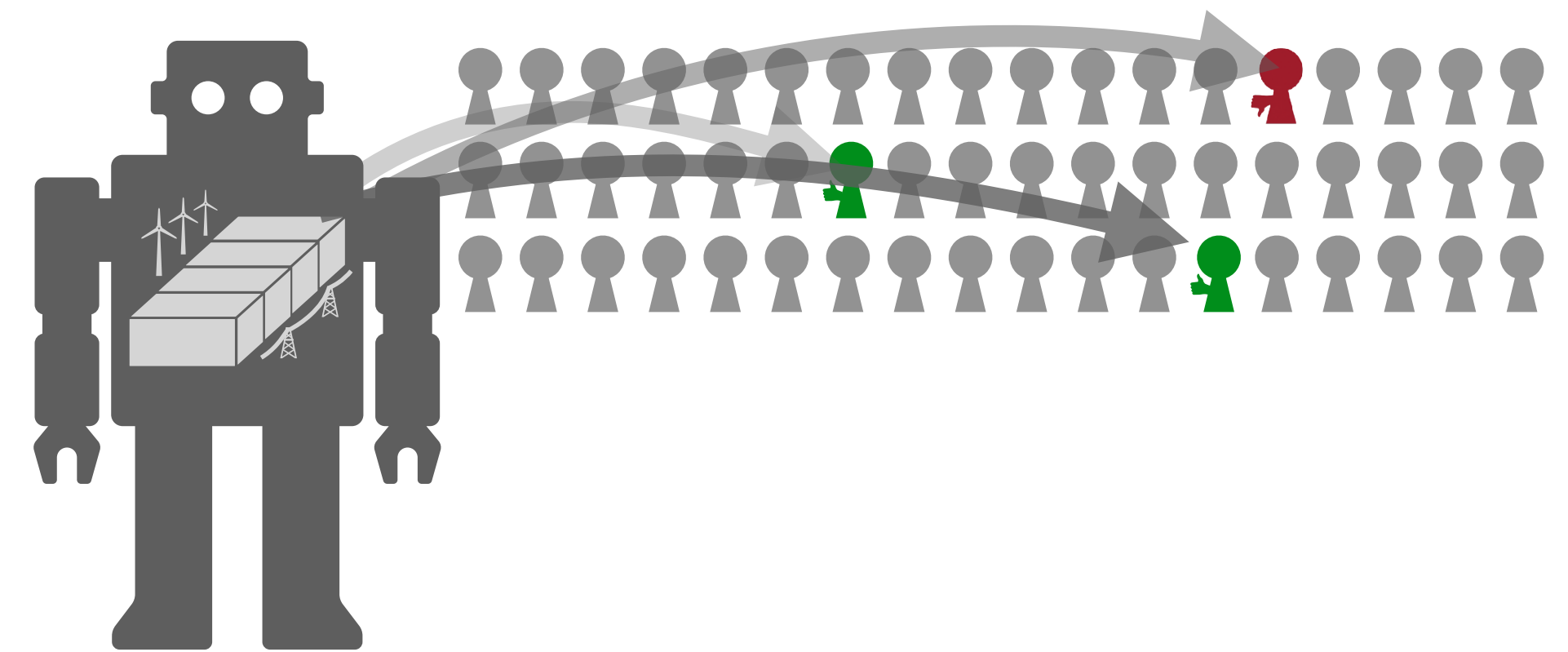
persist even if  $BPP=BQP$ ,  
 $P=NP$ , granted infinite time

# Origin of Advantage

Quantum Algorithm

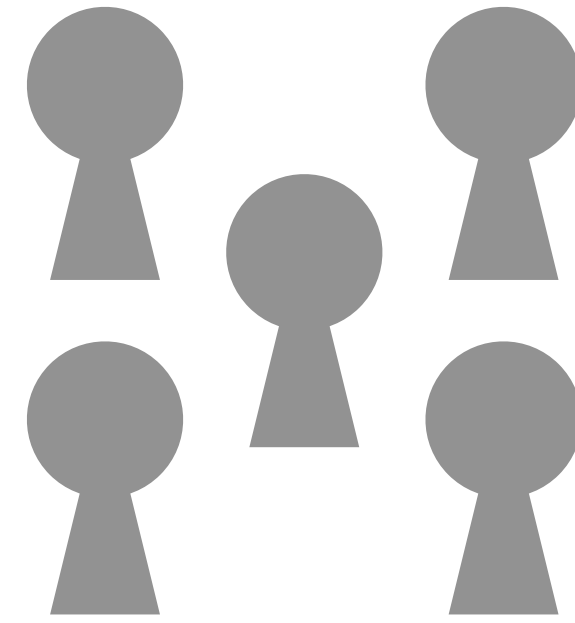
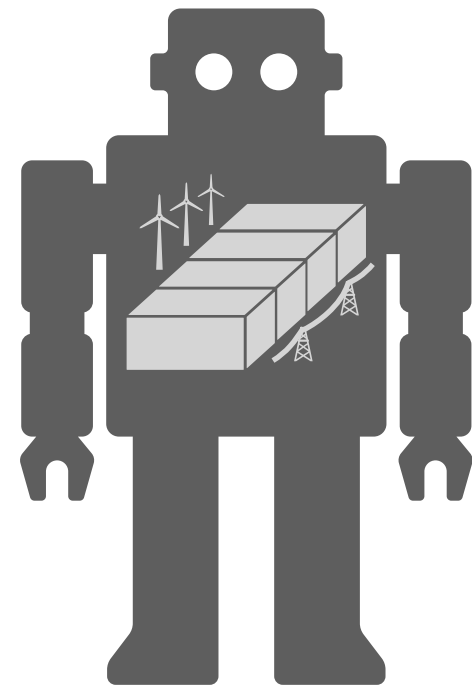


Classical Hardness



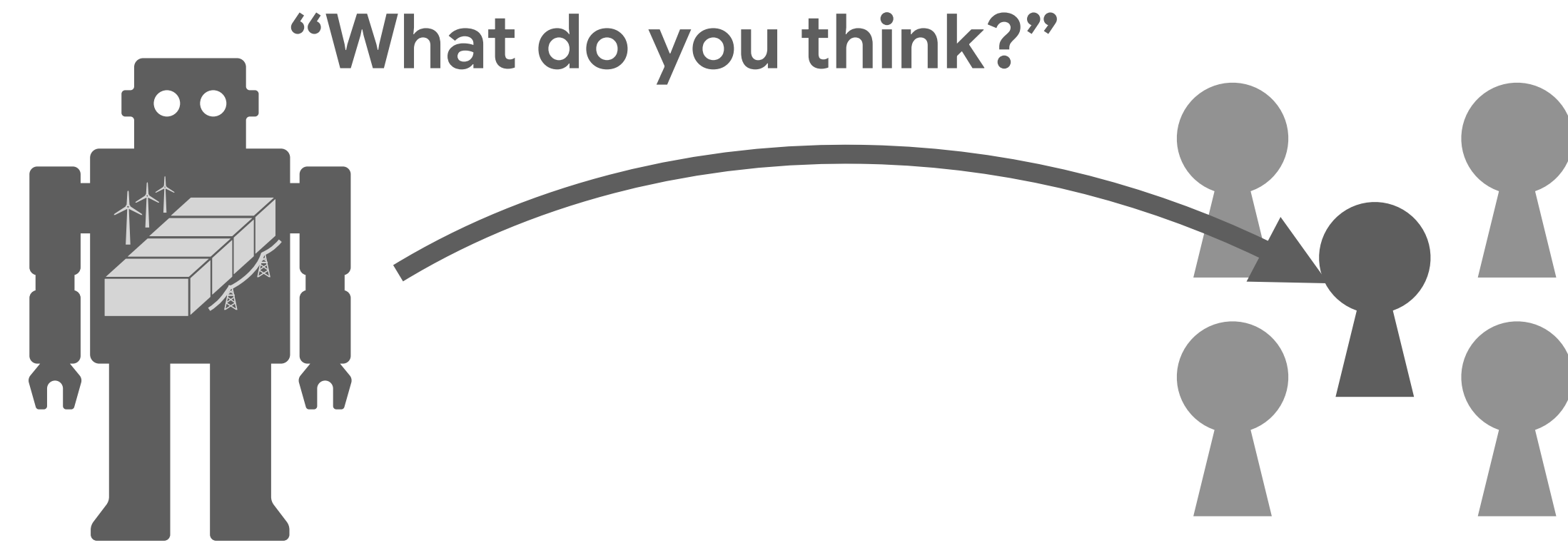
# Quantum advantage?

**Classical ML**



# Quantum advantage?

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# Quantum advantage?

Classical ML



# Quantum advantage?

**Classical ML**



**Quantum ML**

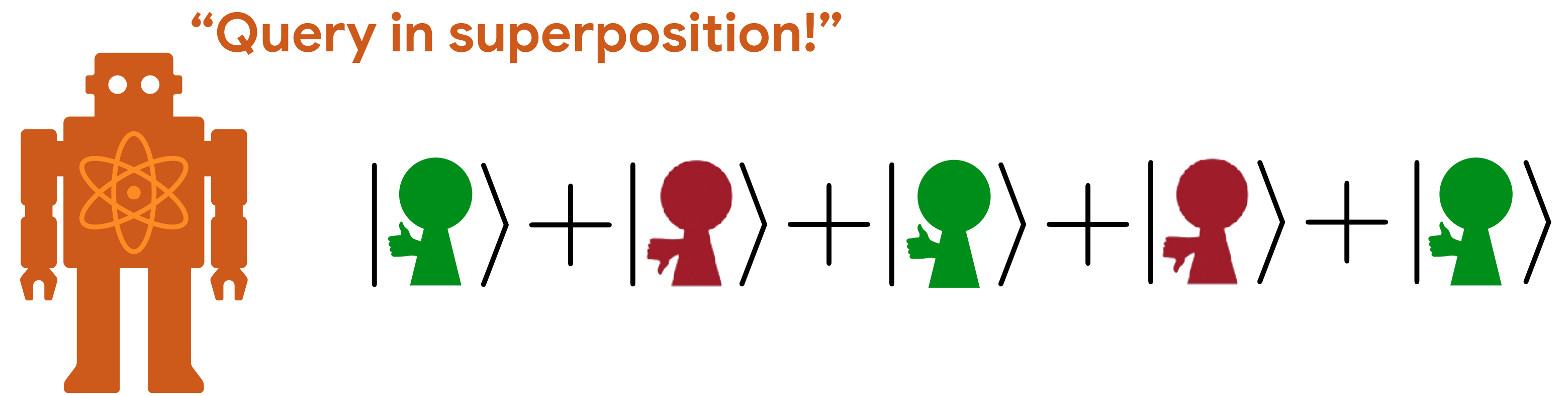


# Quantum advantage?

Classical ML

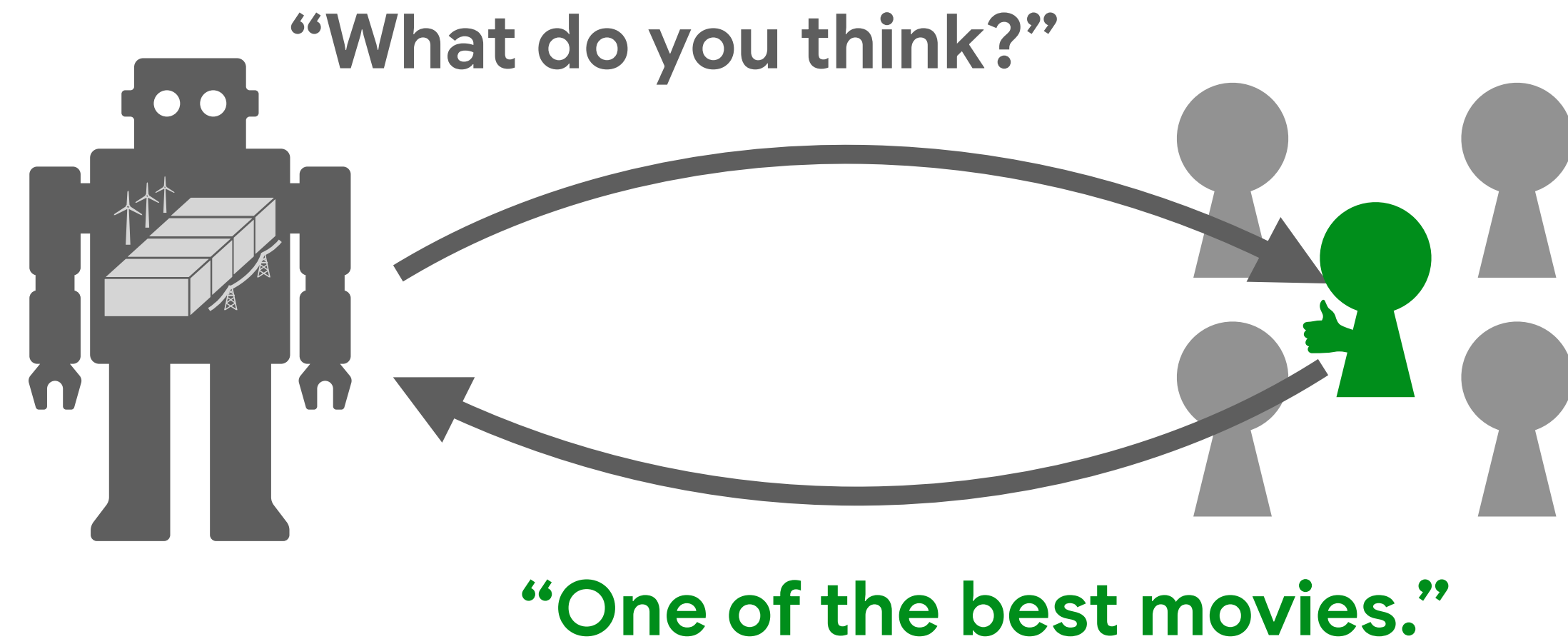


Quantum ML

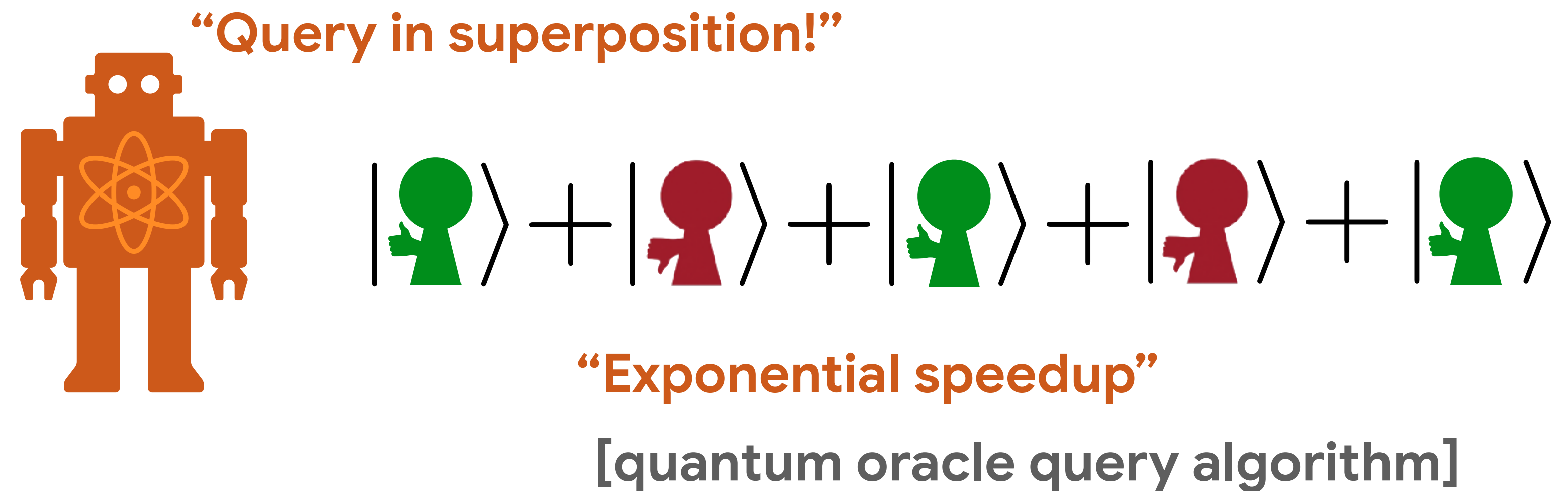


# Quantum advantage?

**Classical ML**

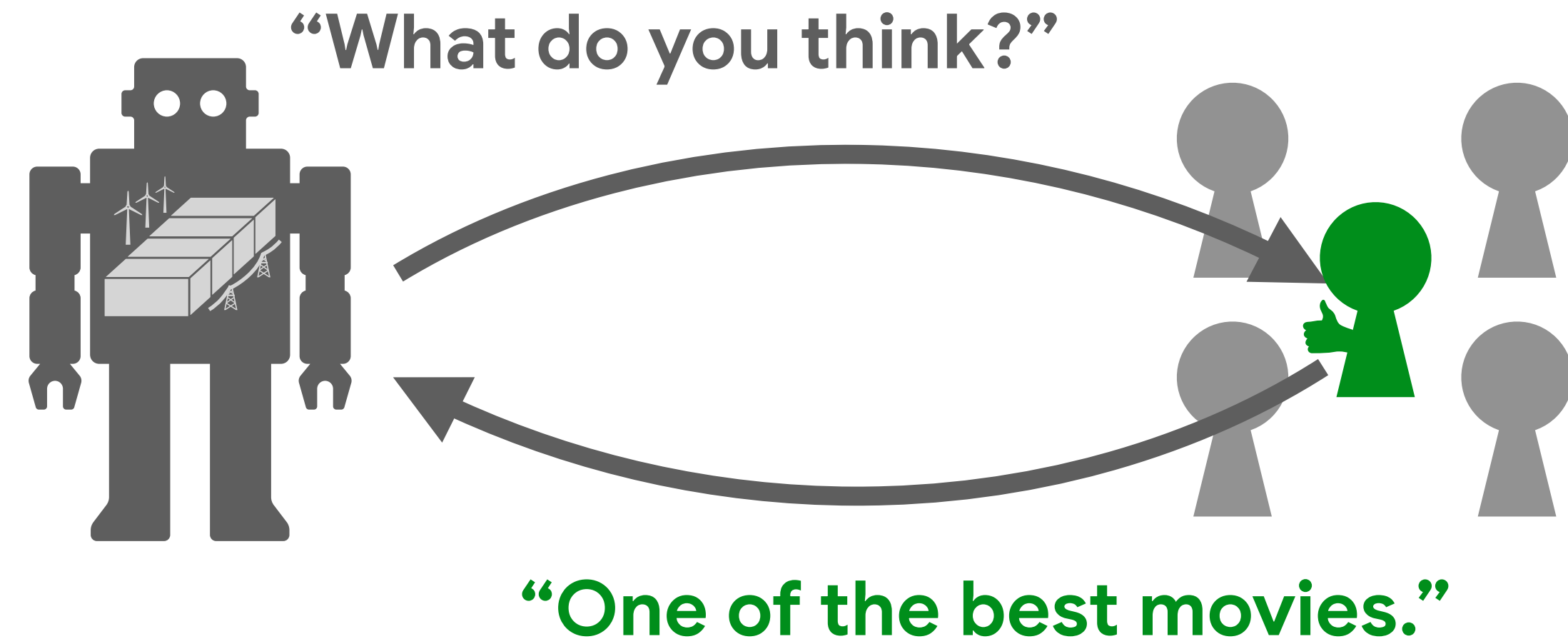


**Quantum ML**



# Quantum advantage?

Classical ML



Quantum ML

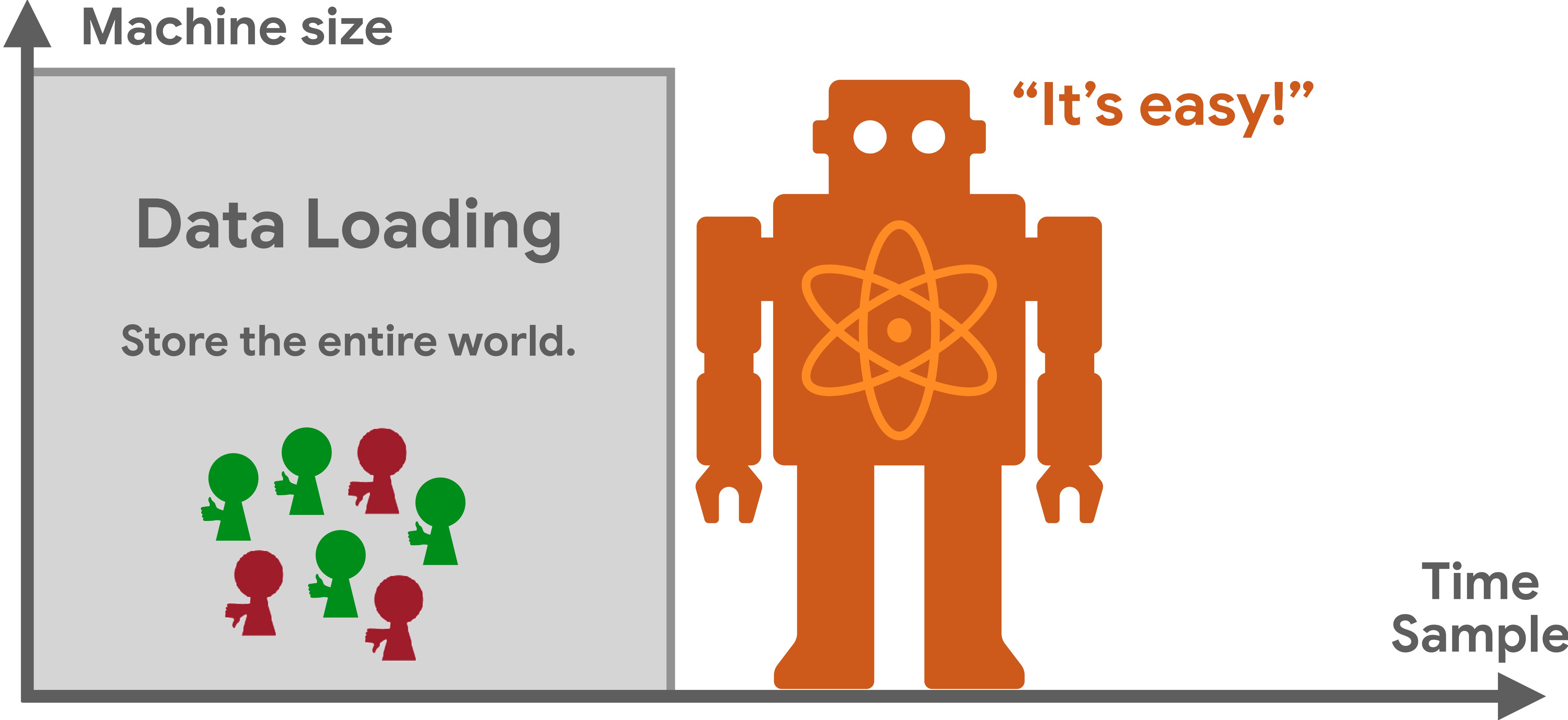


# Challenge

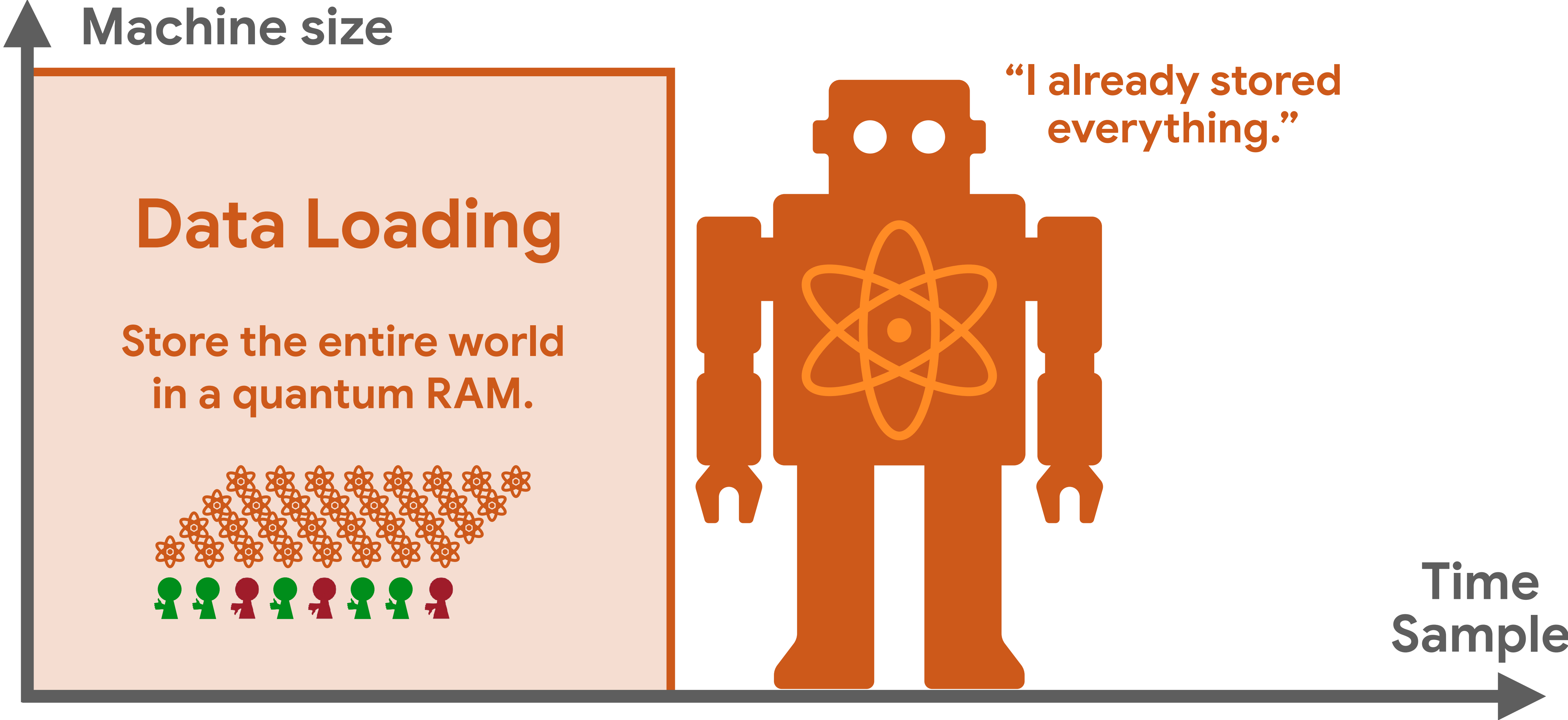
Can we access a classical world  
in quantum superposition?



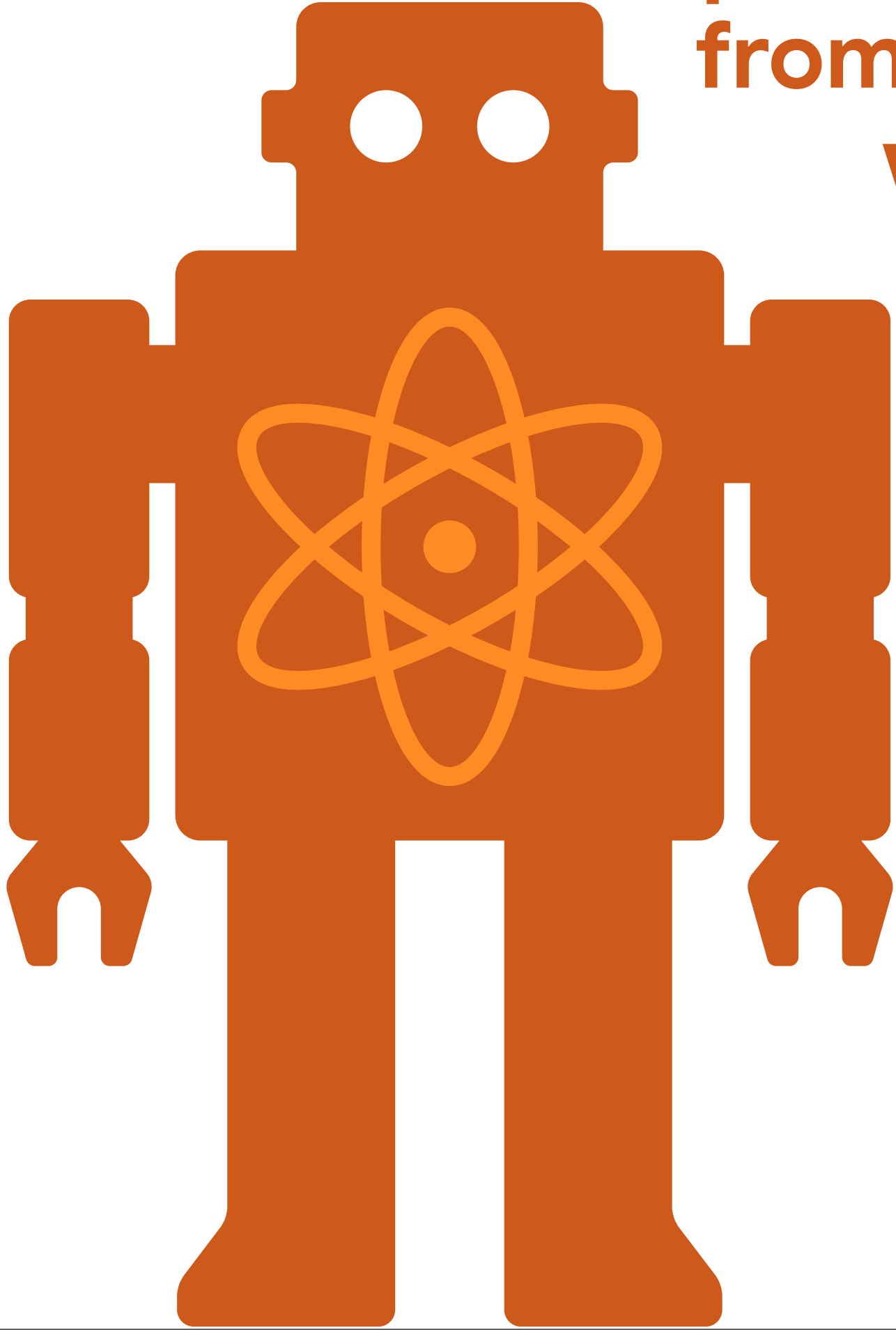
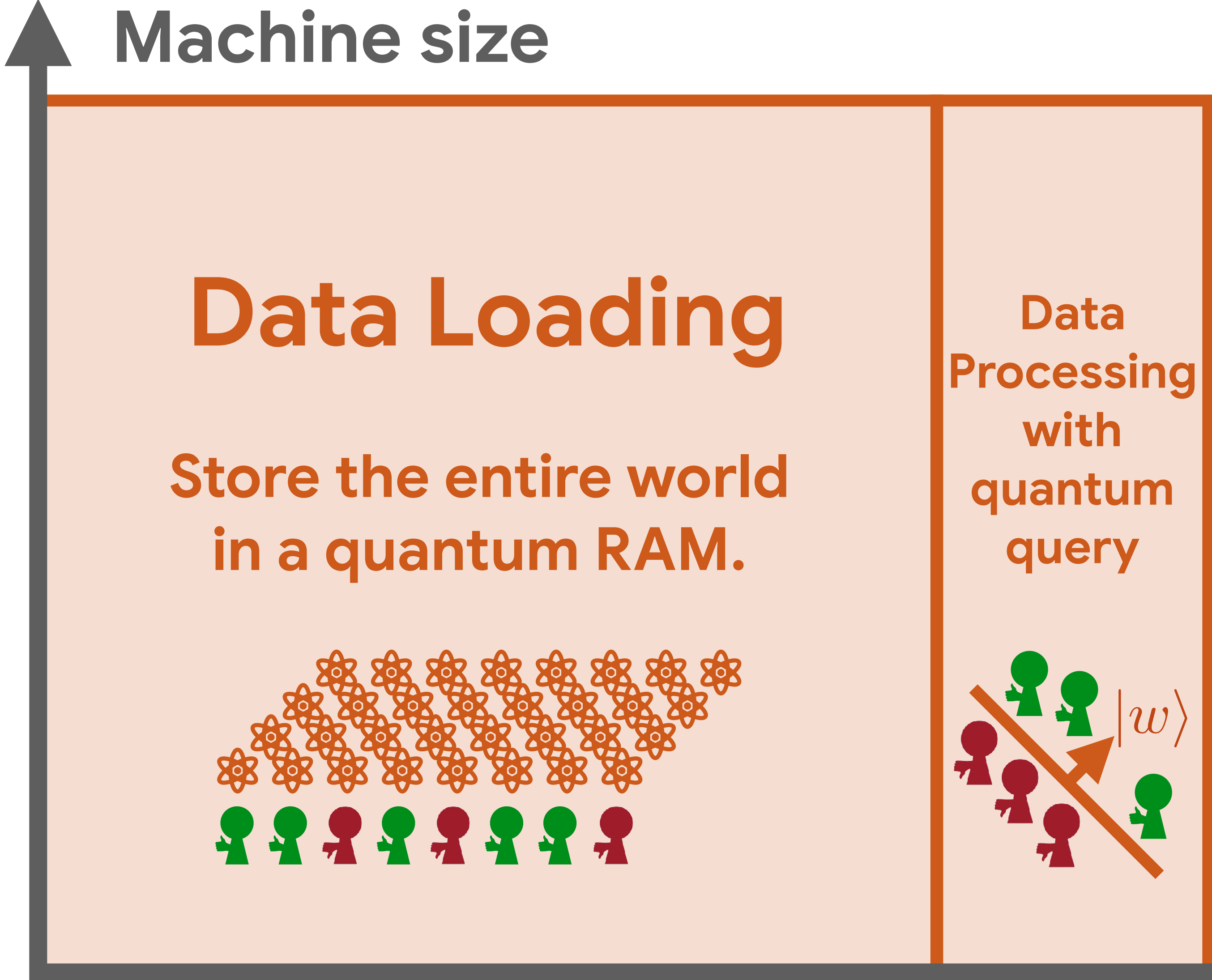
# Attempt: QRAM



# Attempt: QRAM



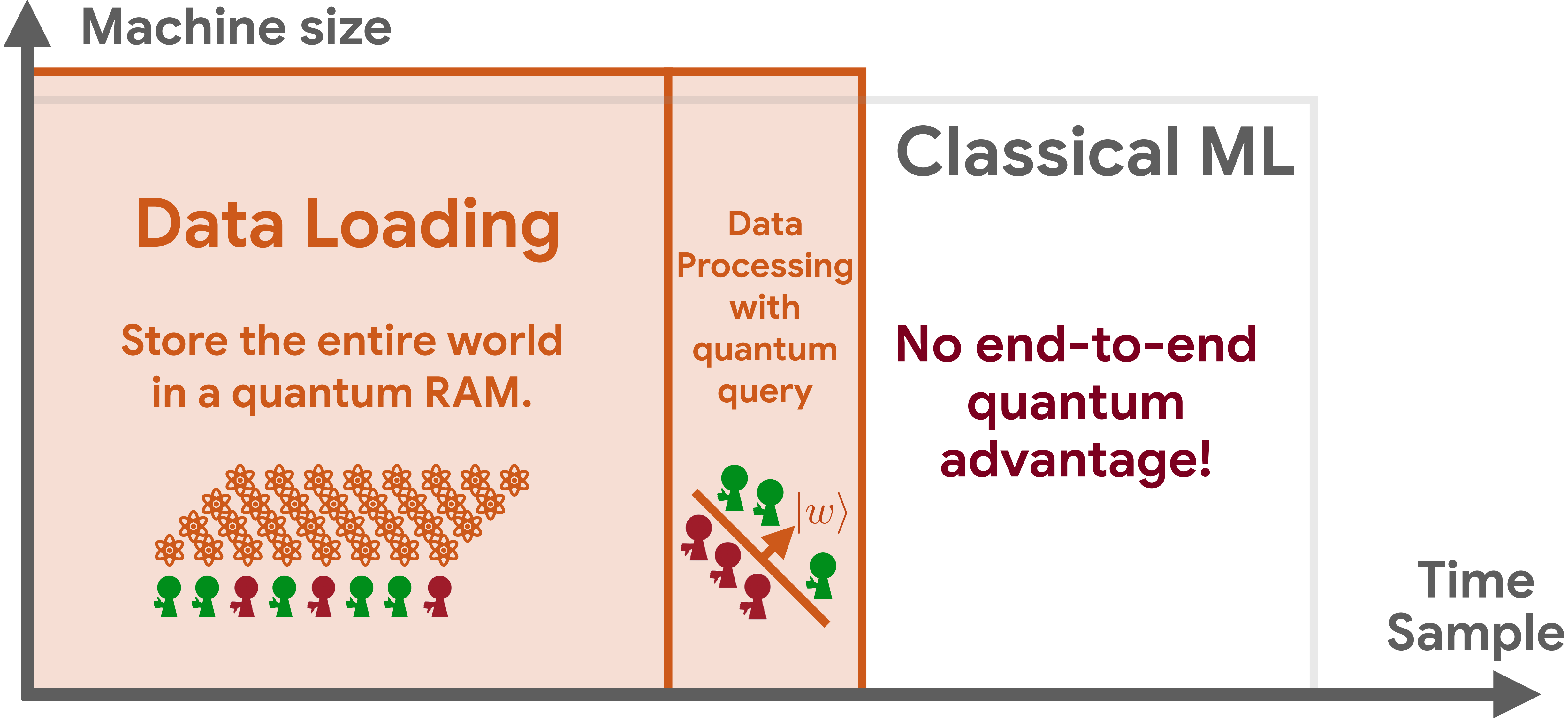
# Attempt: QRAM



“I can create quantum queries from the stored world.”

**Time Sample**

# Attempt: QRAM



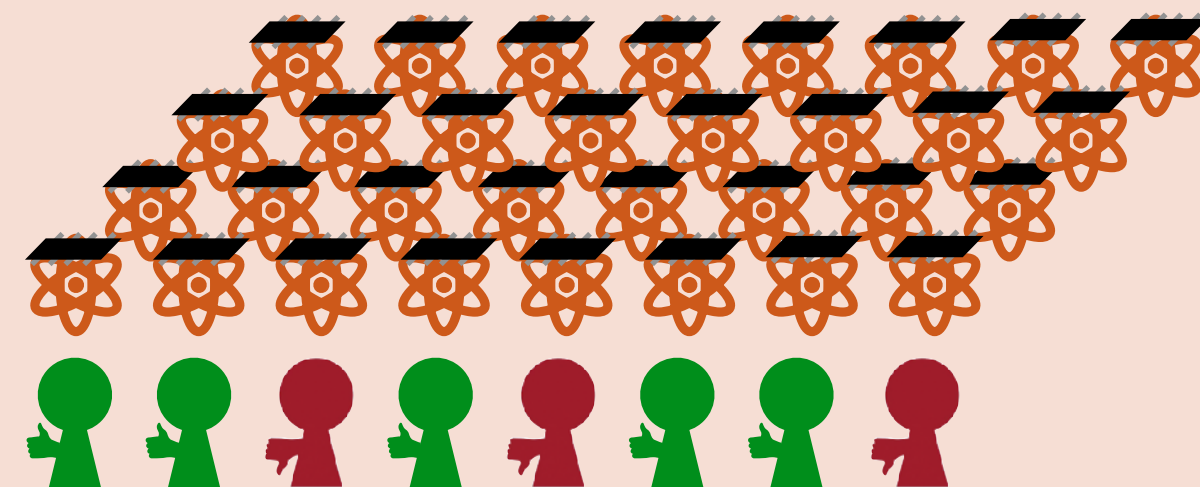
# Attempt: QRAM

Machine size

Classical ML

**Maintaining a coherent  
copy of the entire world.**

Data loading + QEC.



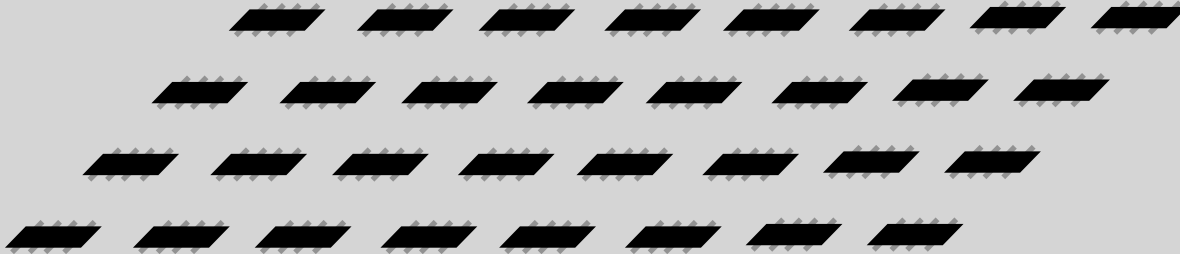
Time  
Sample

# Attempt: QRAM

Machine size

Classical ML

Classical co-processors meant for QEC  
can solve the problem themselves.



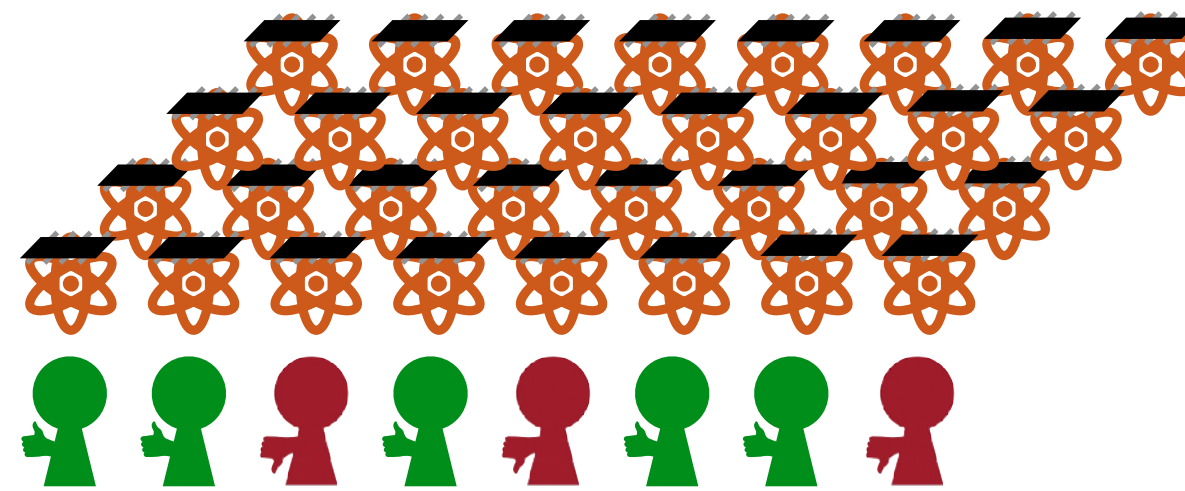
Faster...

Time  
Sample

# Challenge

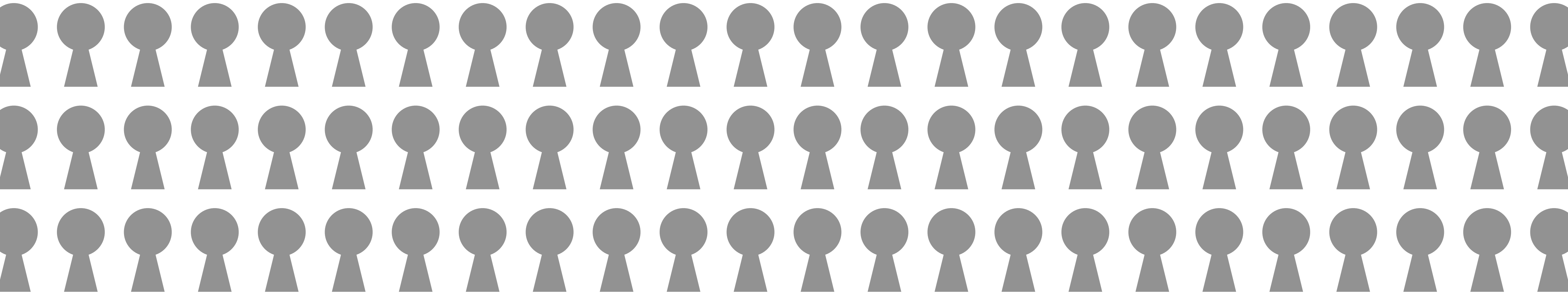
**Can we access a classical world  
in quantum superposition?**

**(without storing the entire world)**

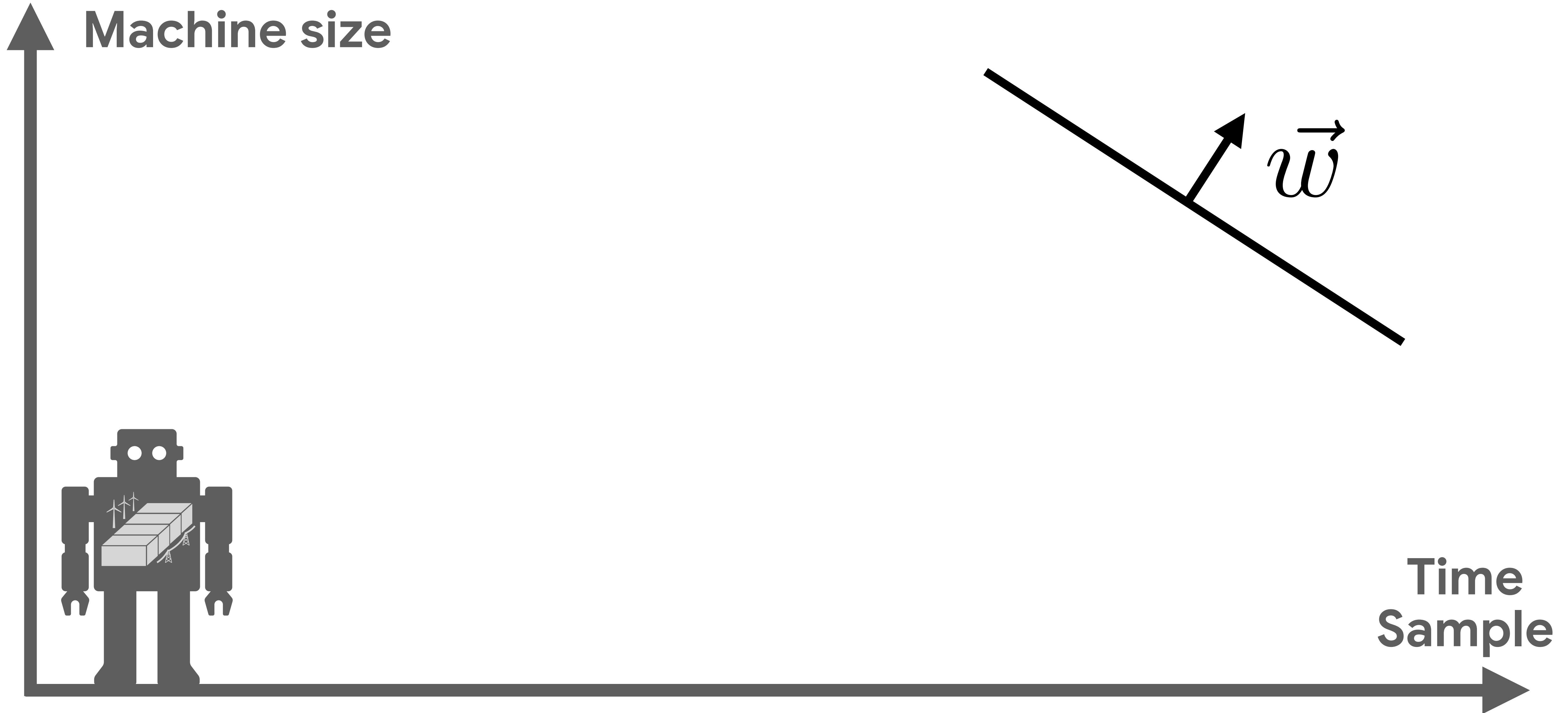


# Challenge

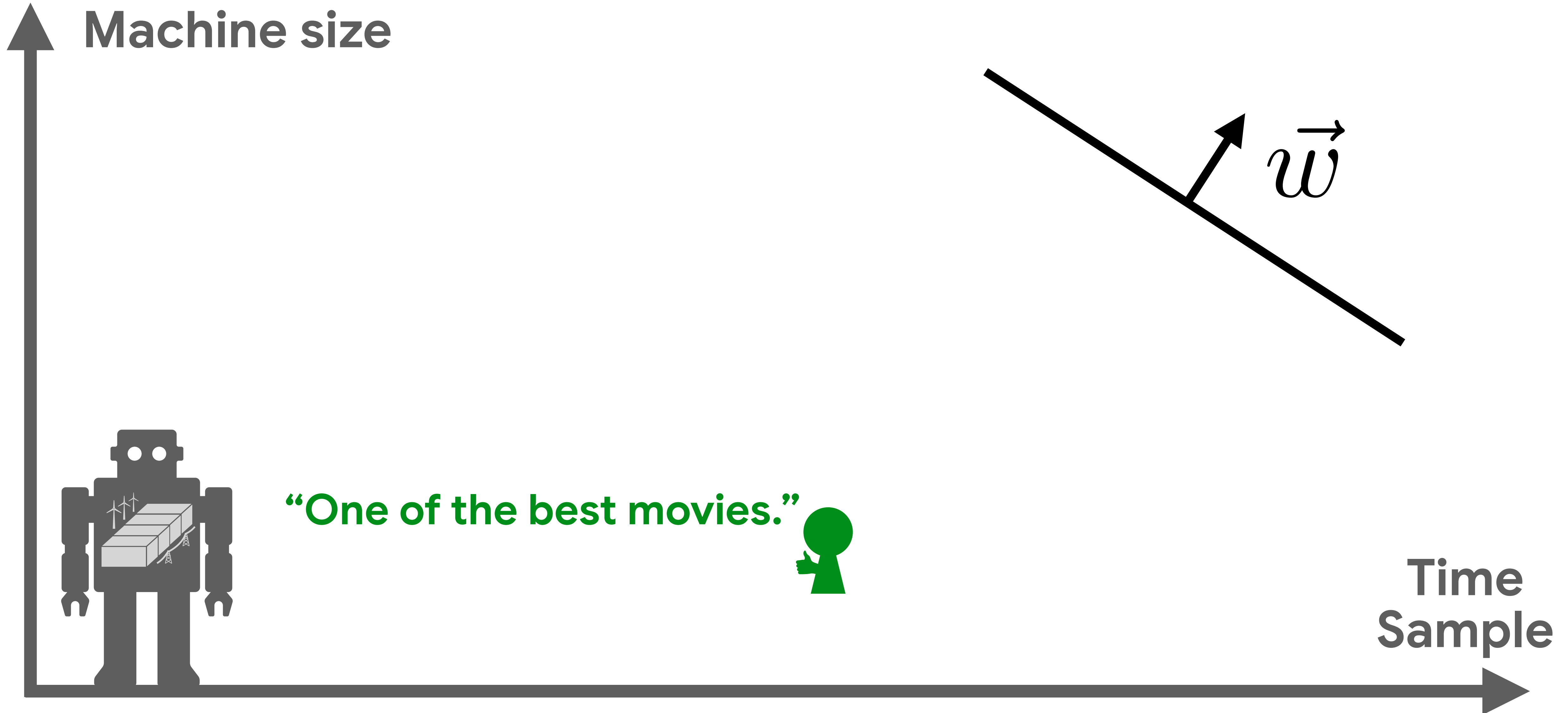
How to do ML without  
storing the entire dataset?



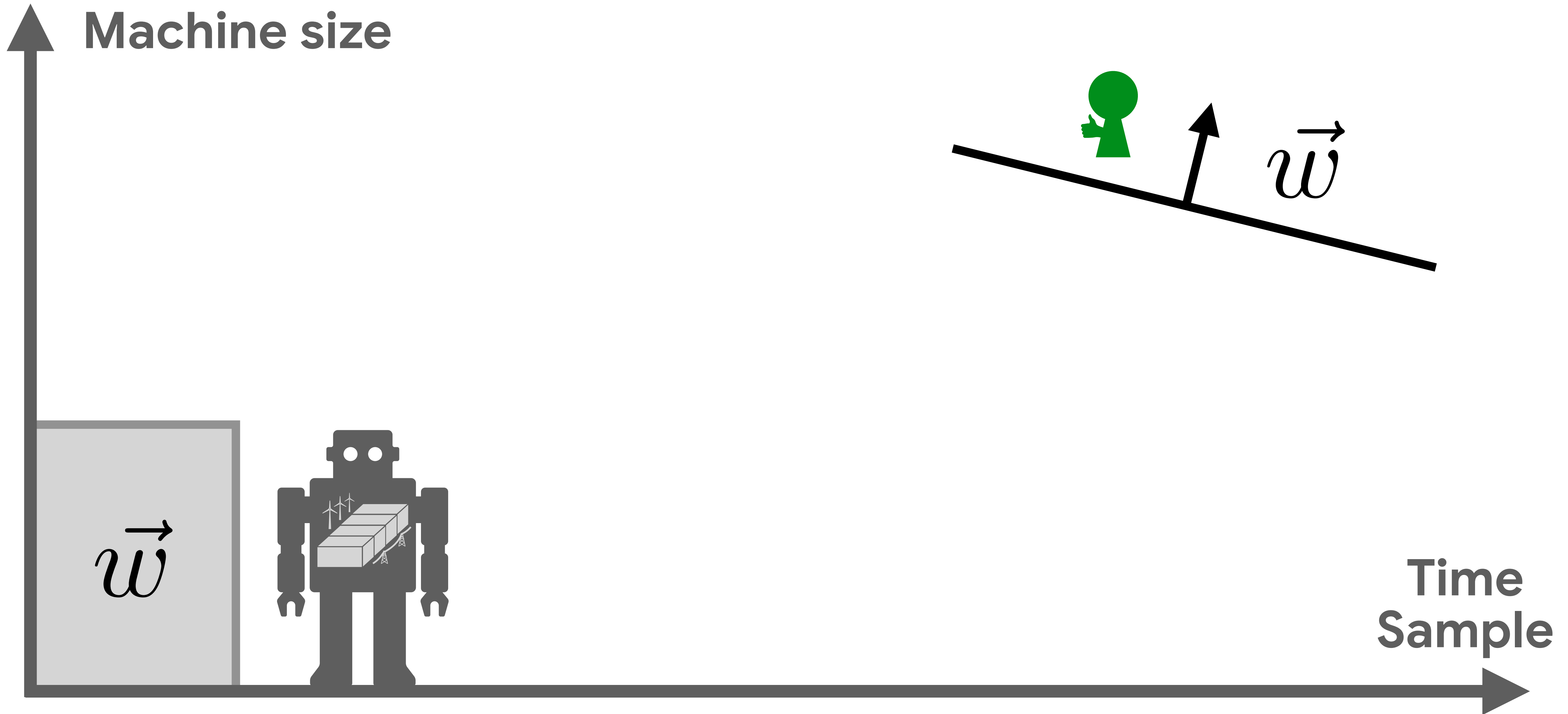
# Train a model on the fly



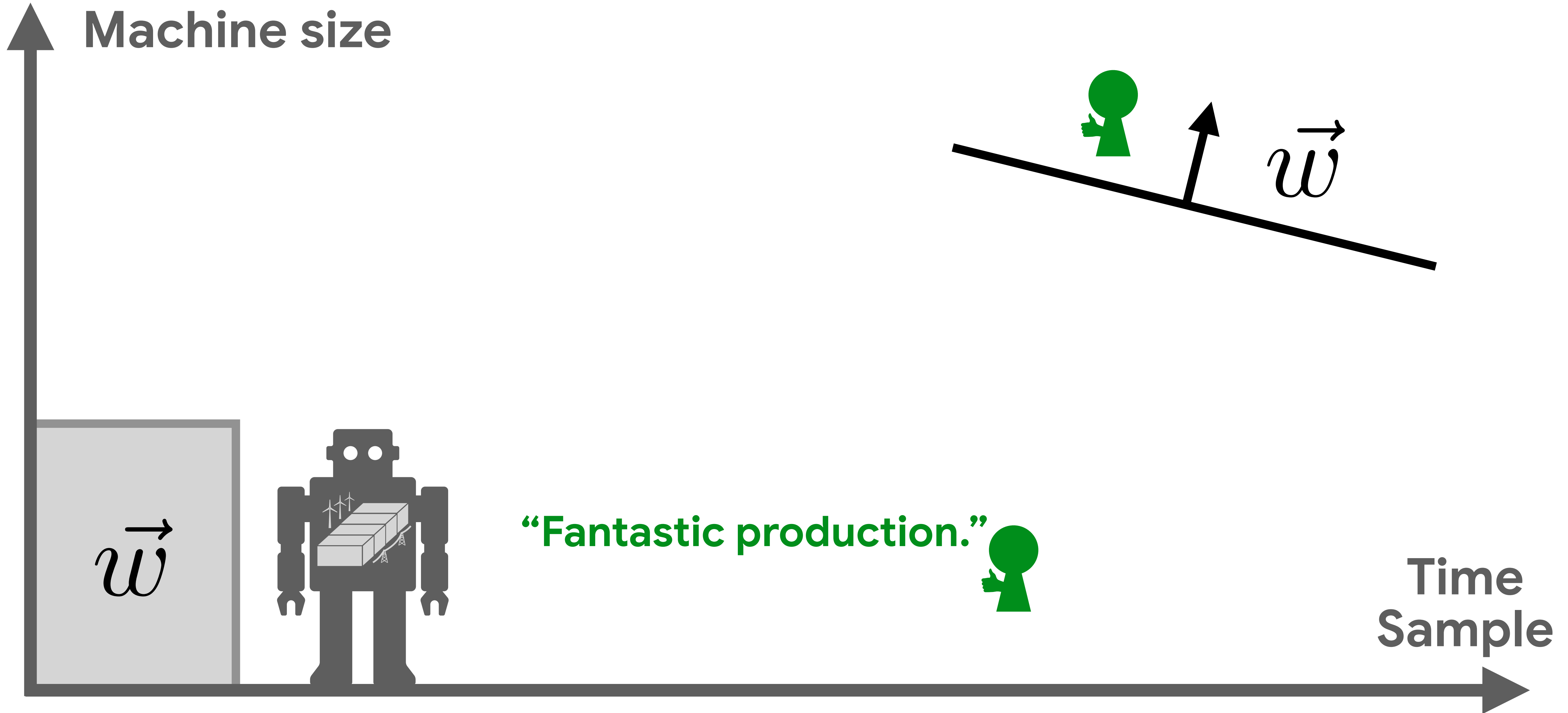
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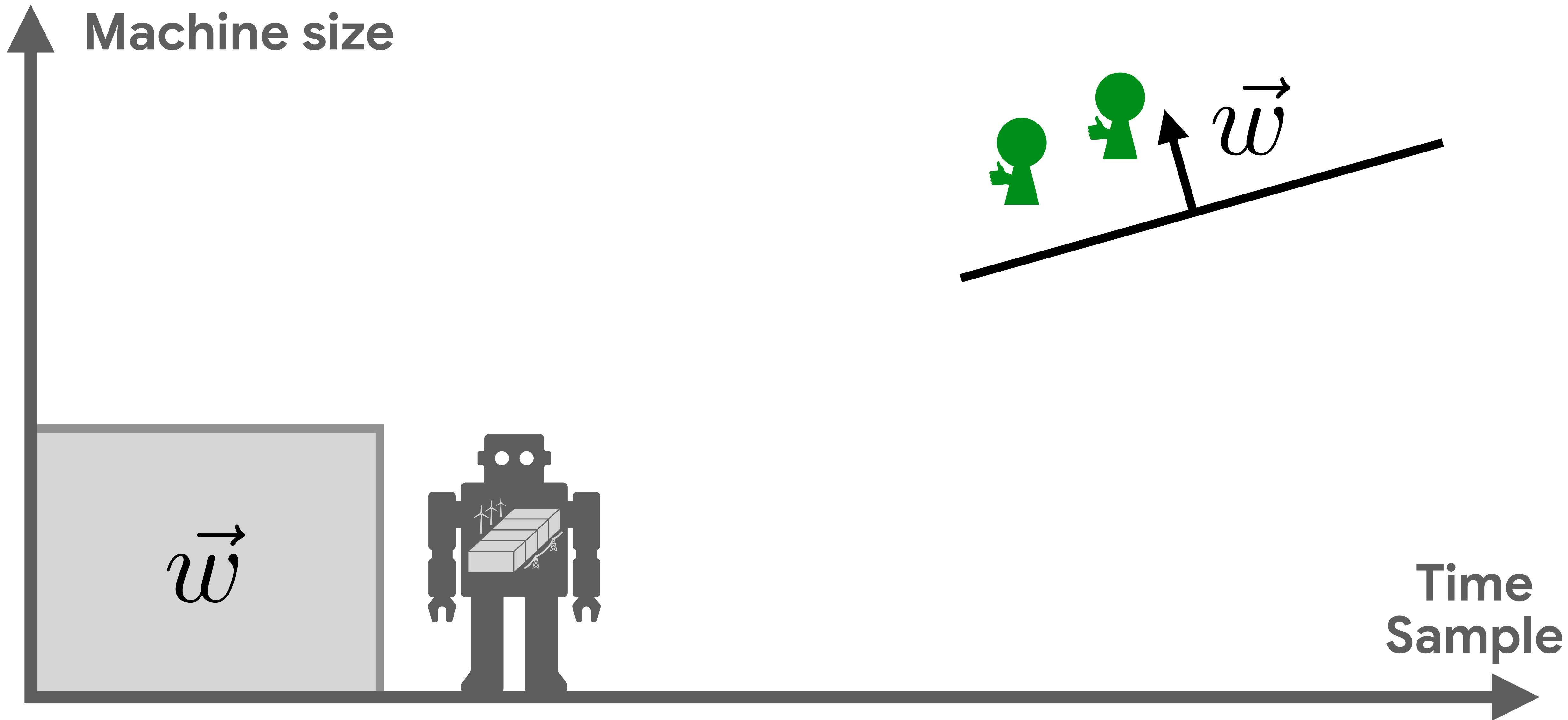
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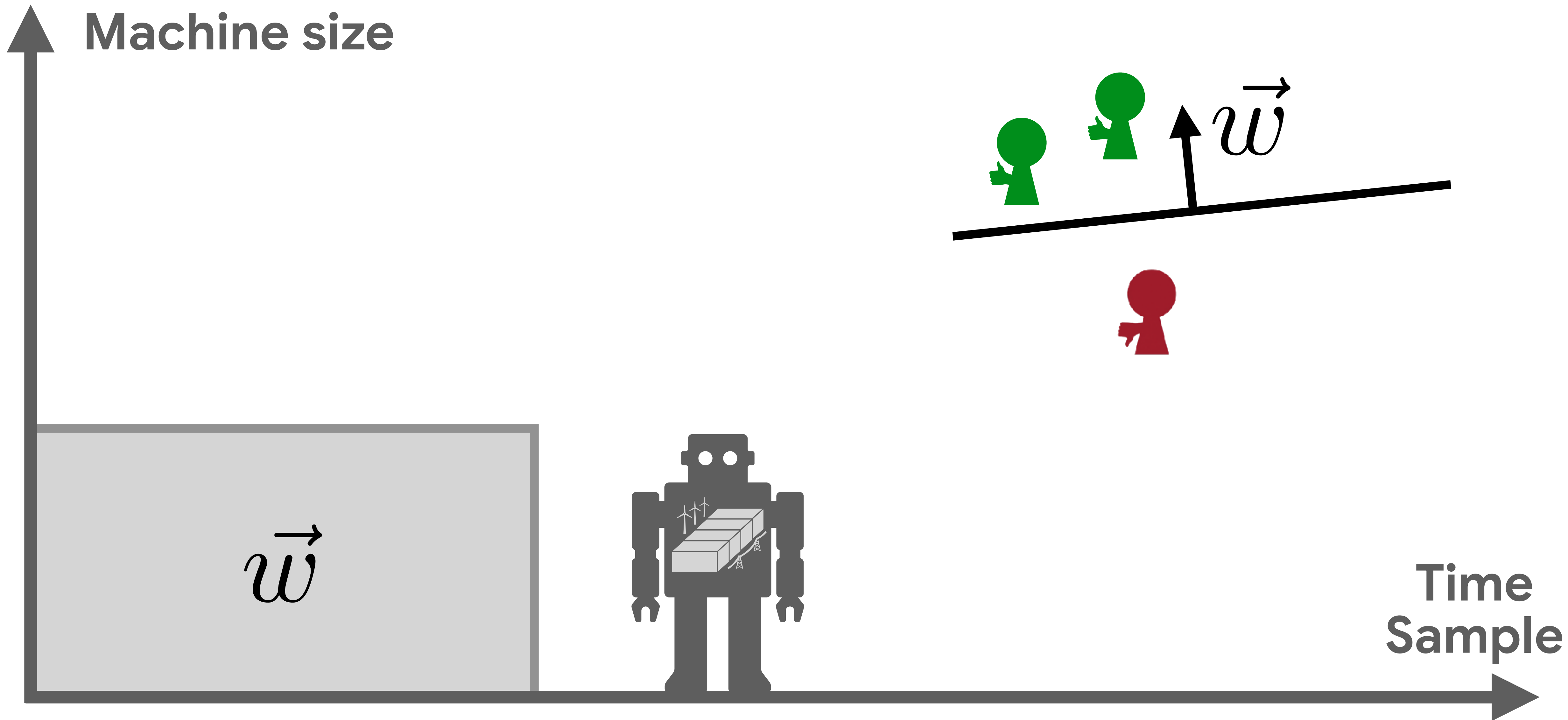
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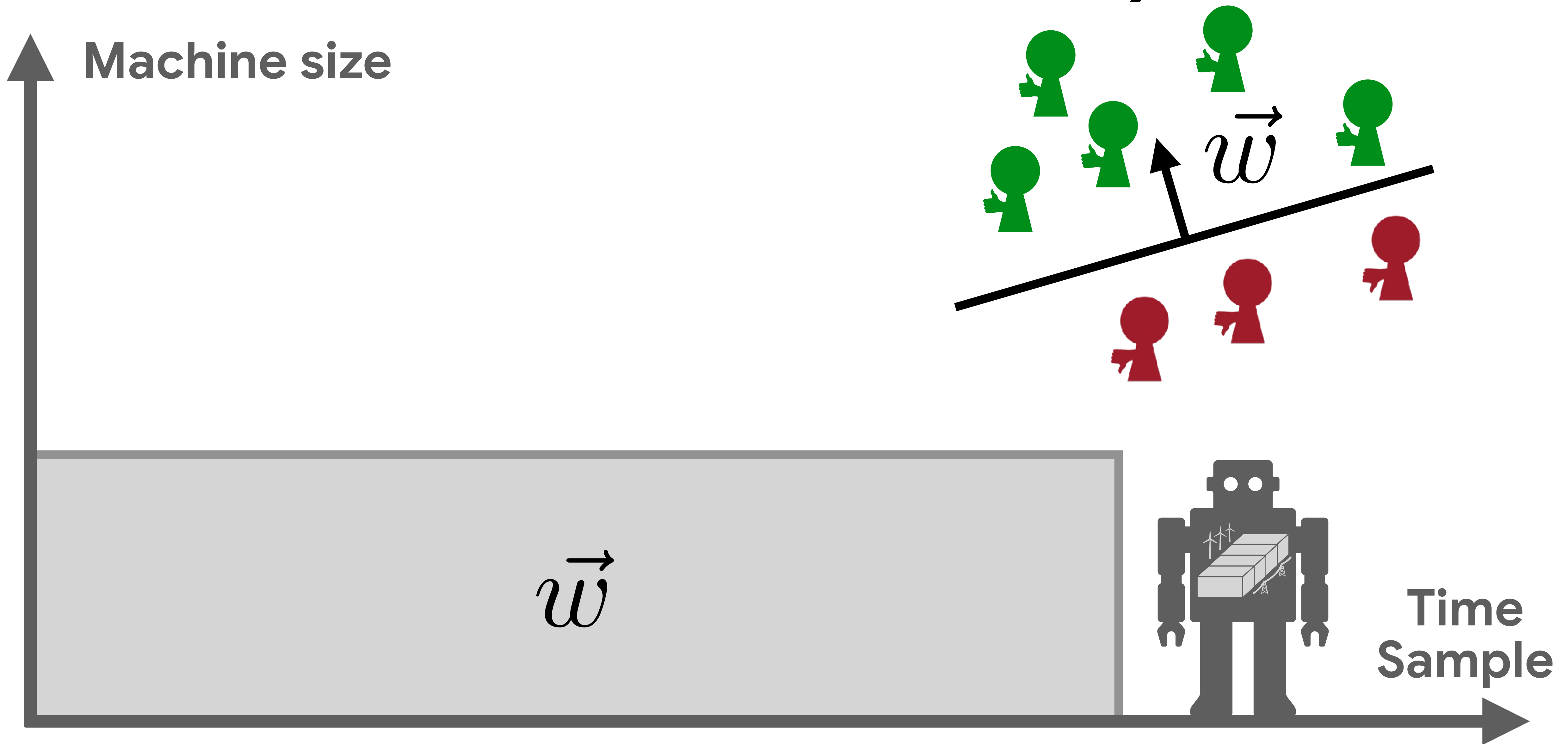
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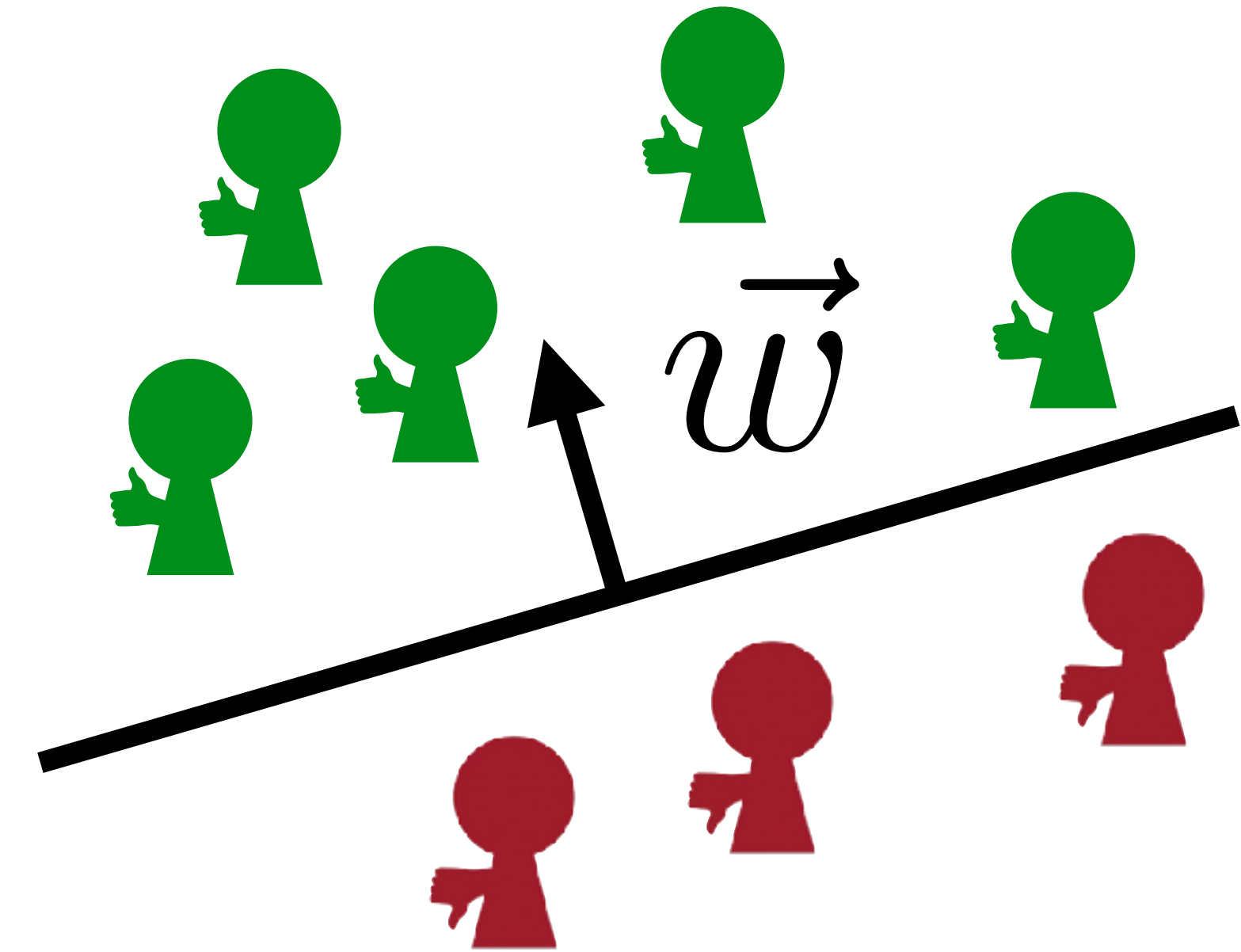


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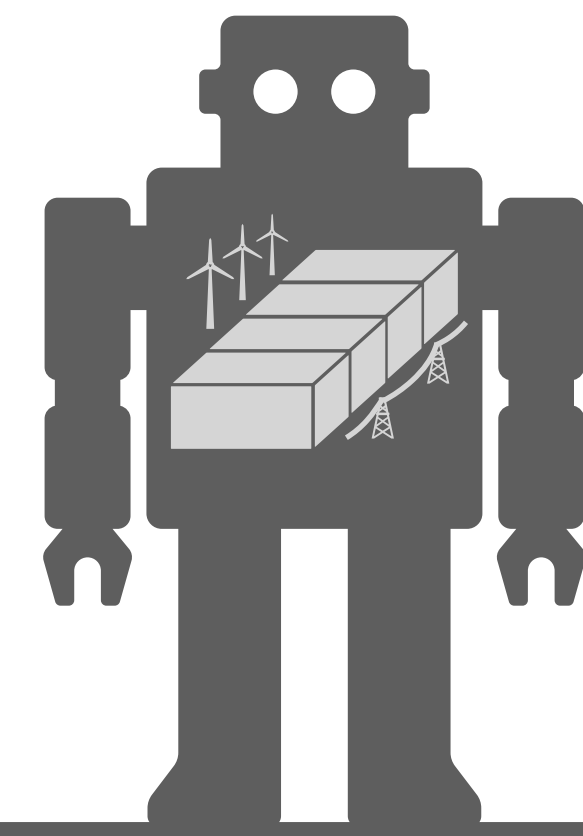
Machine size



Process data on the fly

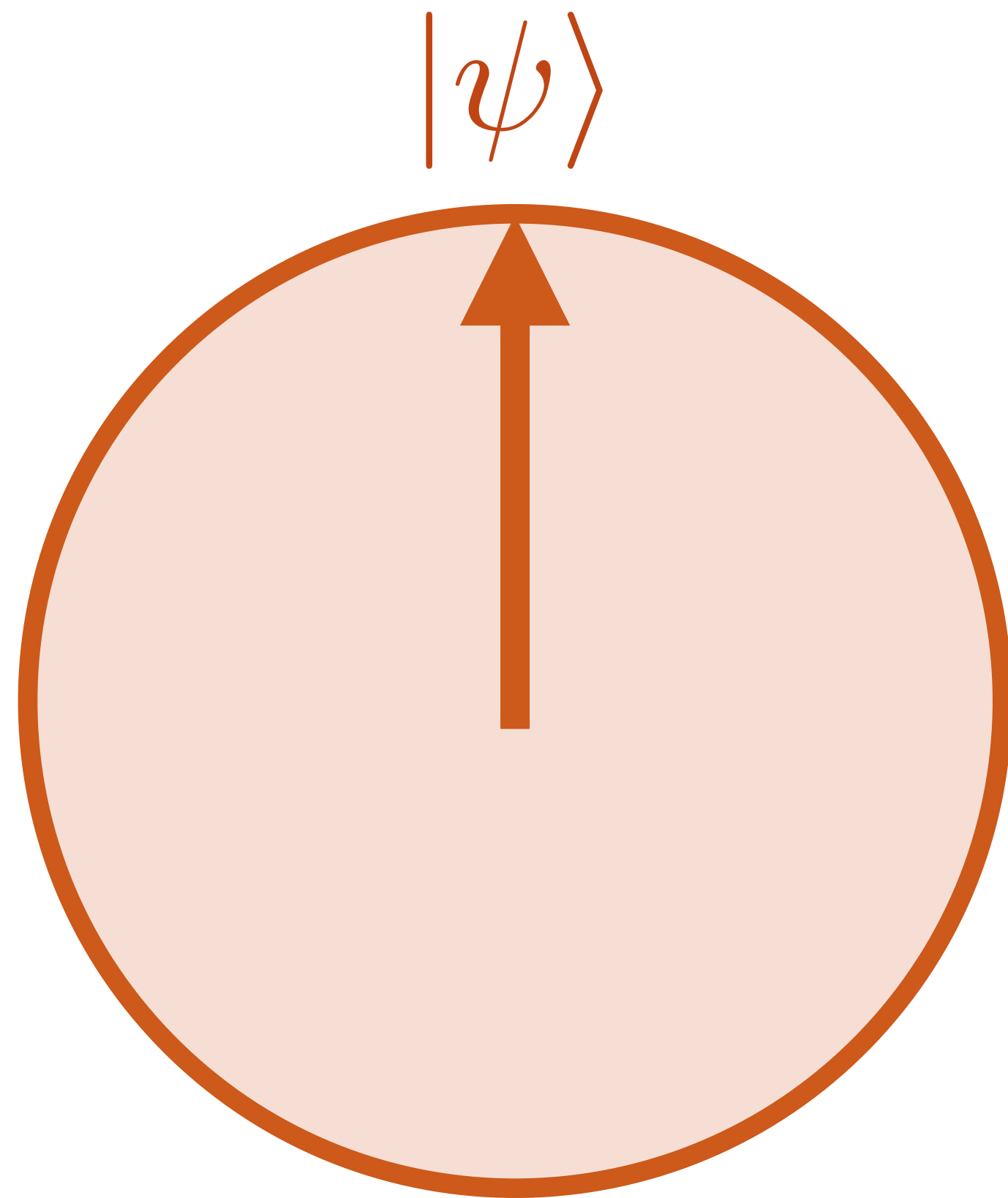
Incrementally update a world model.

[streaming, sketching, online learning]



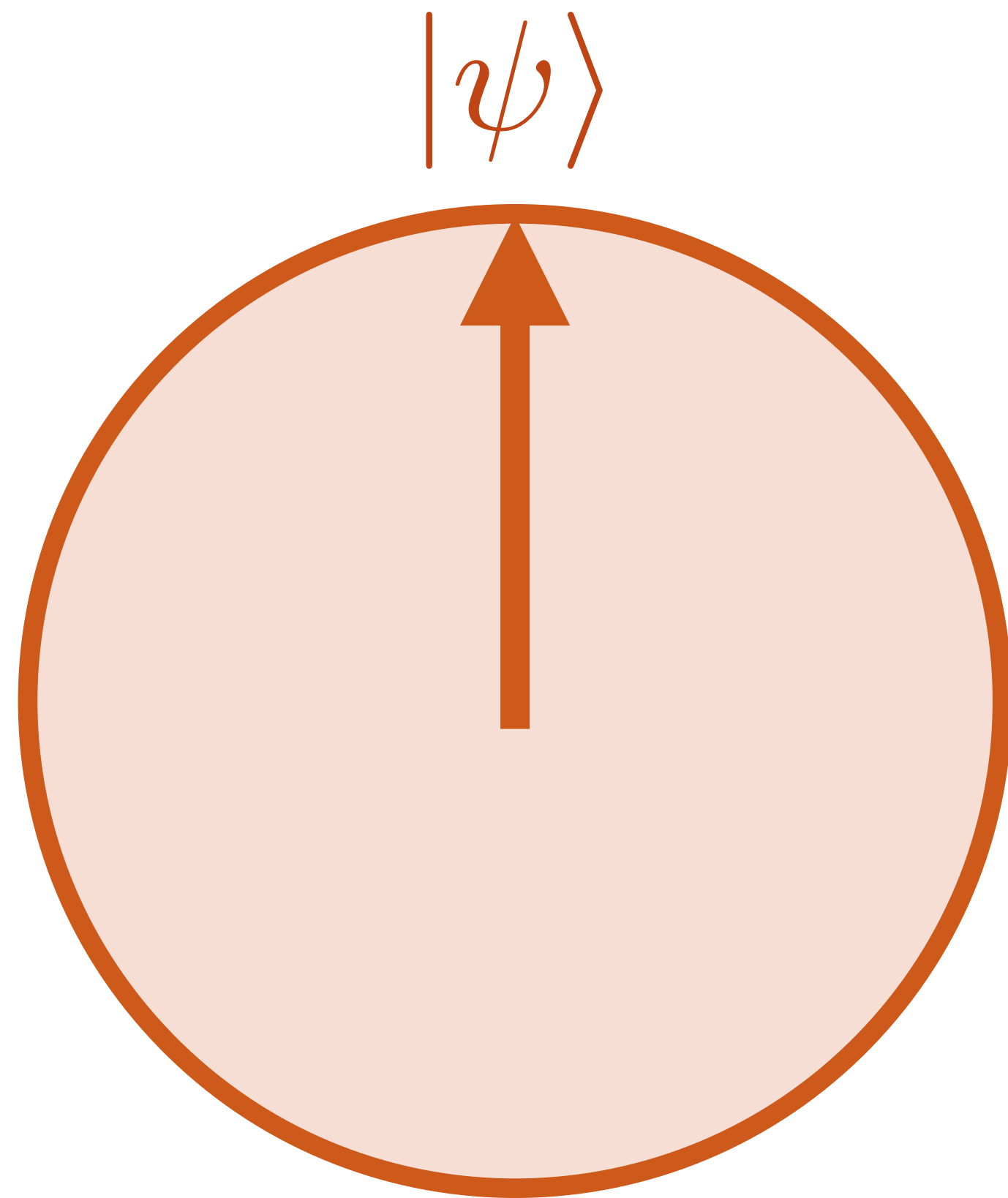
Time Sample

# Quantum oracle sketching



**Maintain a quantum model**

# Quantum oracle sketching



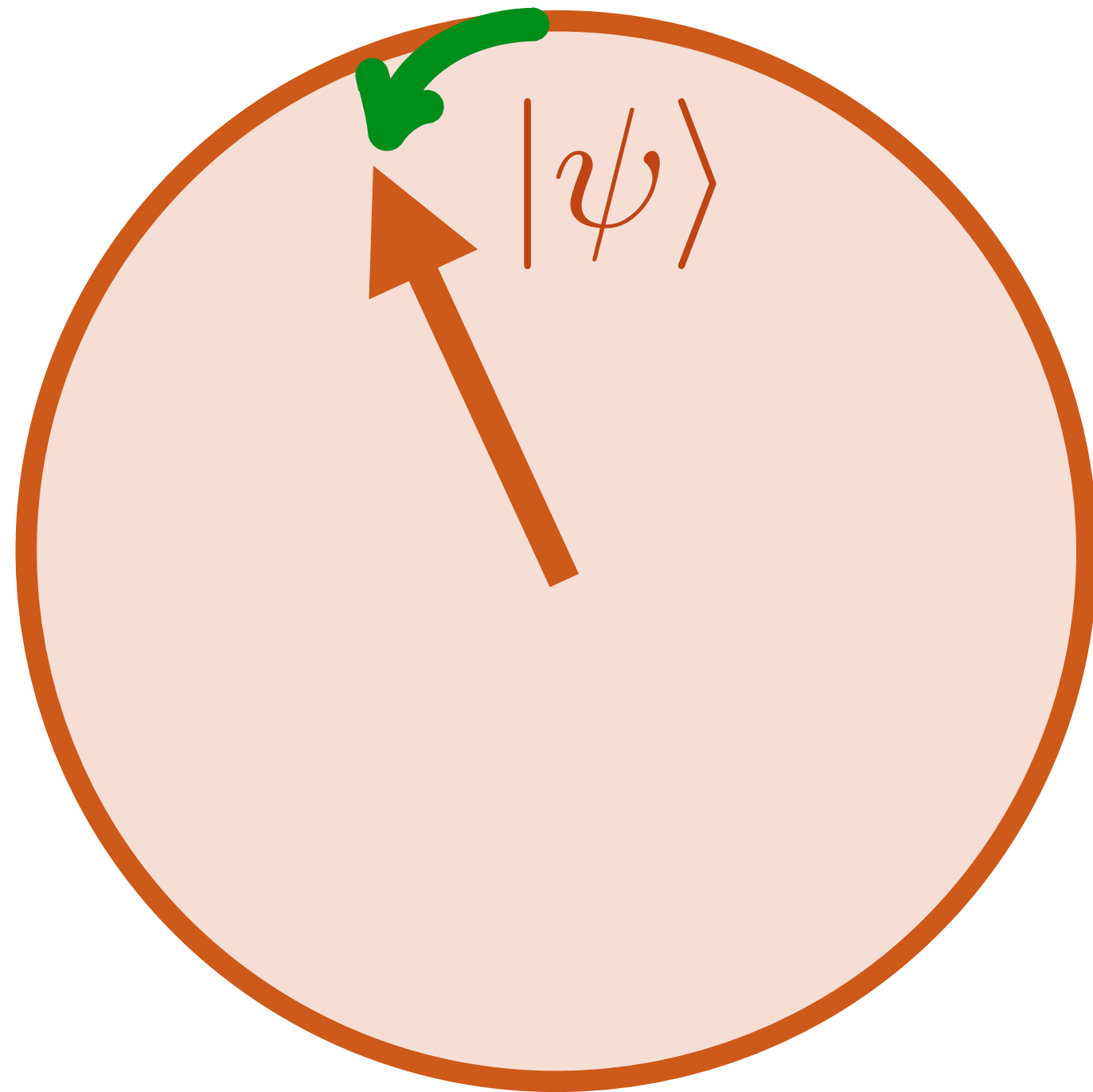
Maintain a quantum model

For each data sample  $z$

“One of the best movies.”



# Quantum oracle sketching



Maintain a quantum model

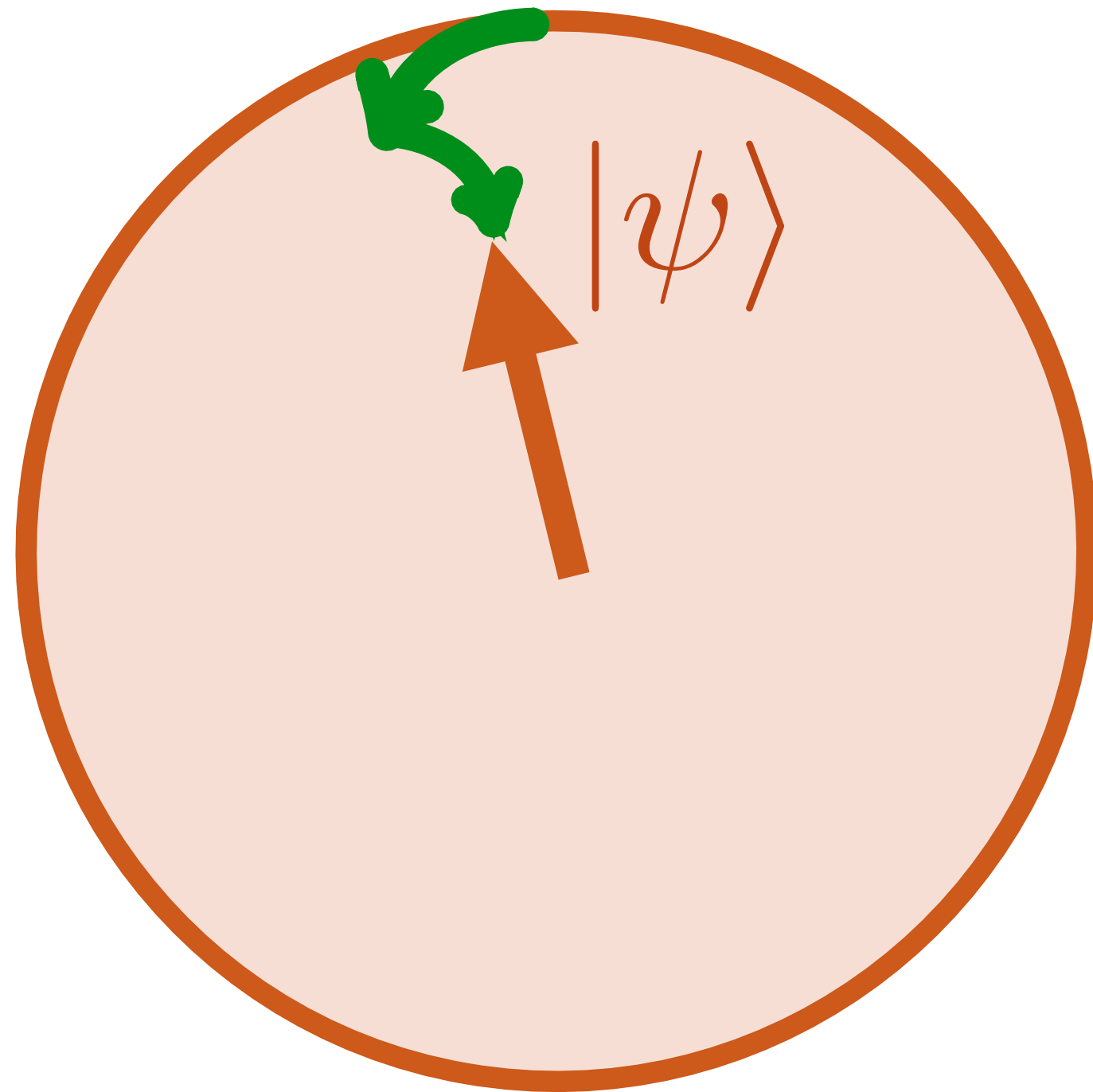
For each data sample  $z$

apply a small quantum  
rotation  $V_z$

“One of the best movies.”



# Quantum oracle sketching



Maintain a quantum model

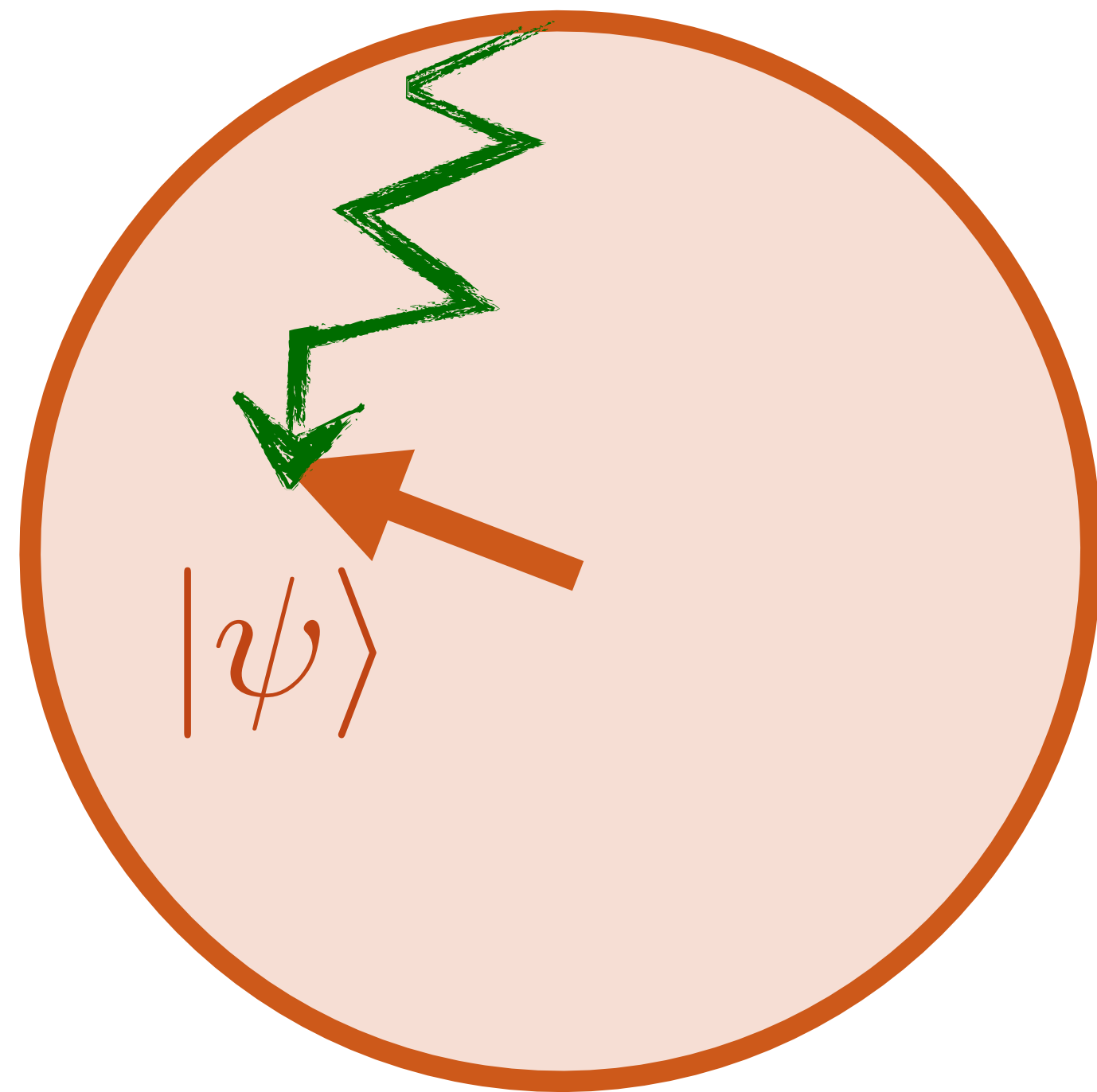
For each data sample  $z$

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Repeat for  $M$  samples

# Quantum oracle sketching



Maintain a quantum model

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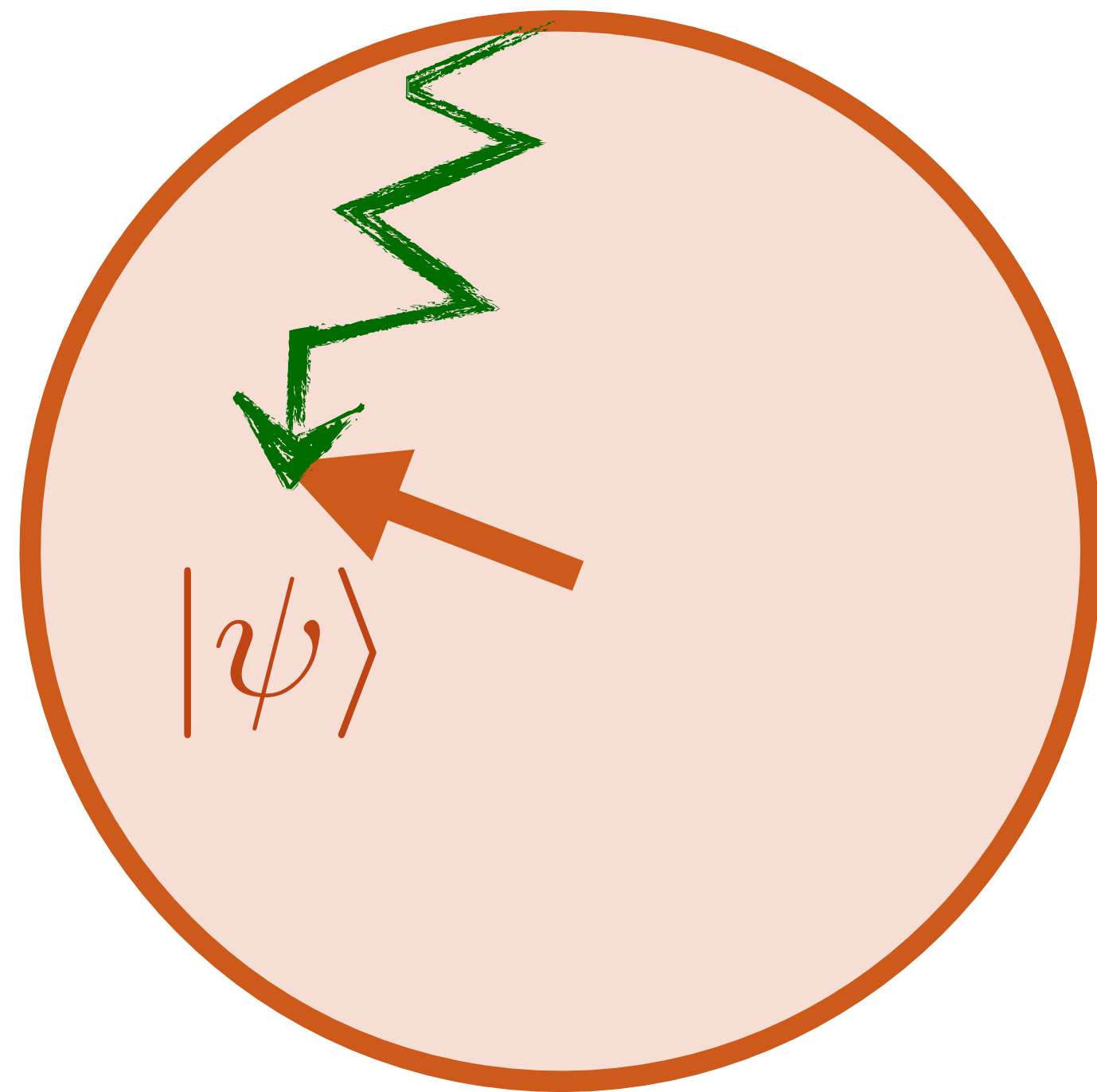
apply a small quantum  
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Repeat for  $M$  samples

a random gate sequence

$$V_{z_M} \cdots V_{z_1}$$

# Quantum oracle sketching



a random gate sequence

$$V_{z_M} \cdots V_{z_1}$$

Maintain a quantum model

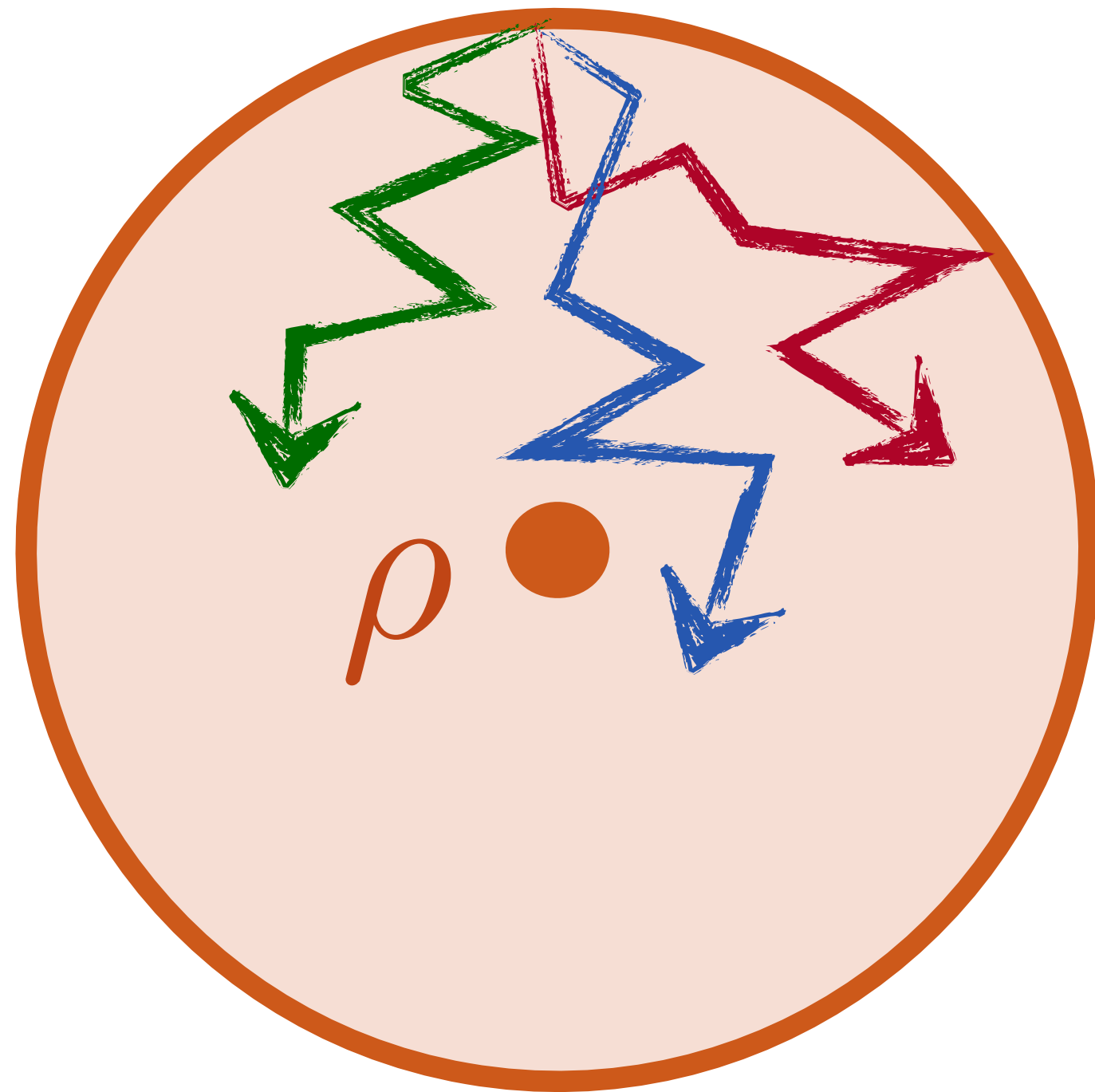
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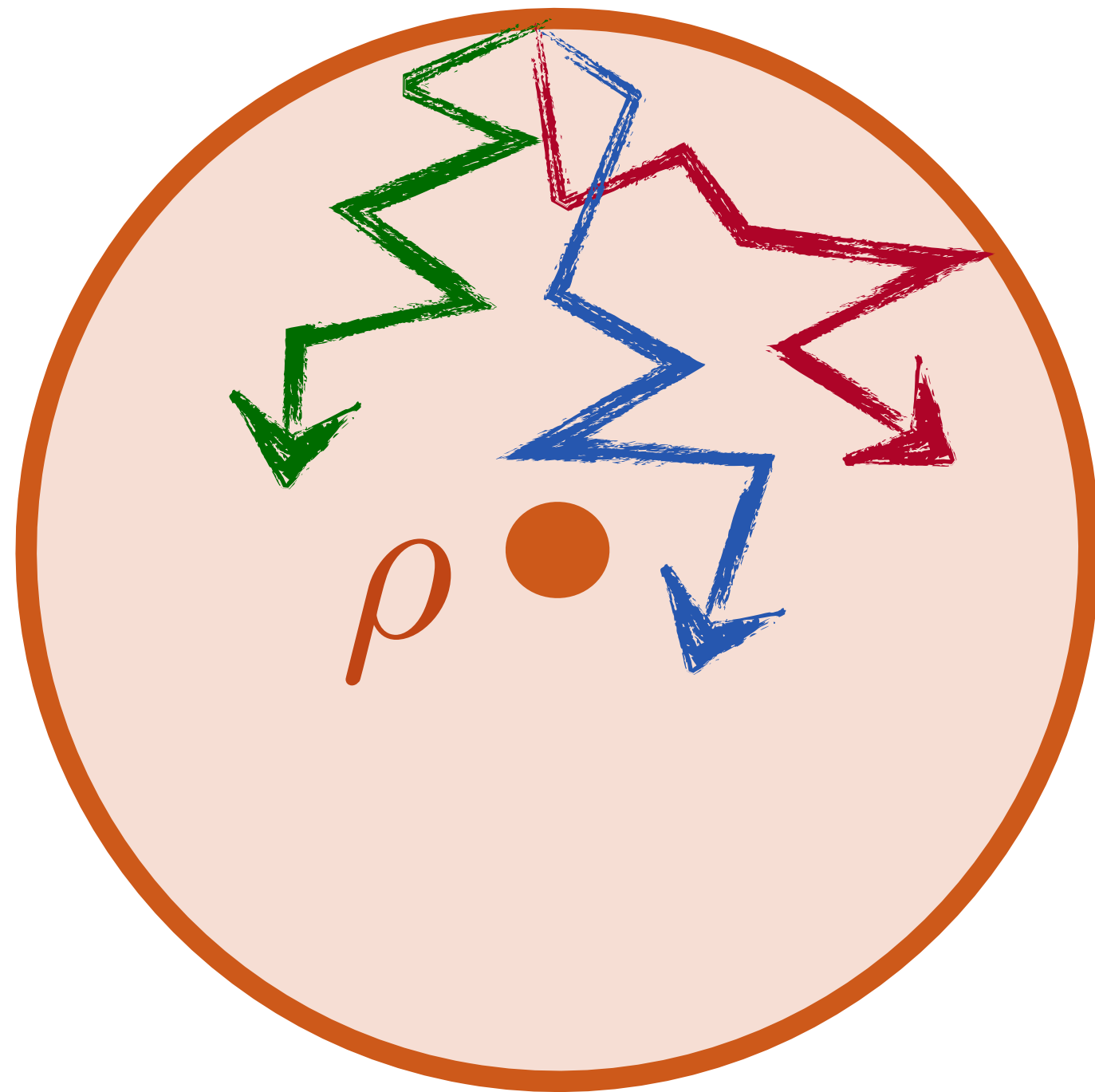
$\approx$  a quantum oracle query

# Quantum oracle sketching



Data randomness leads to decoherence for generic rotations  $V_{z_M} \cdots V_{z_1}$

# Quantum oracle sketching

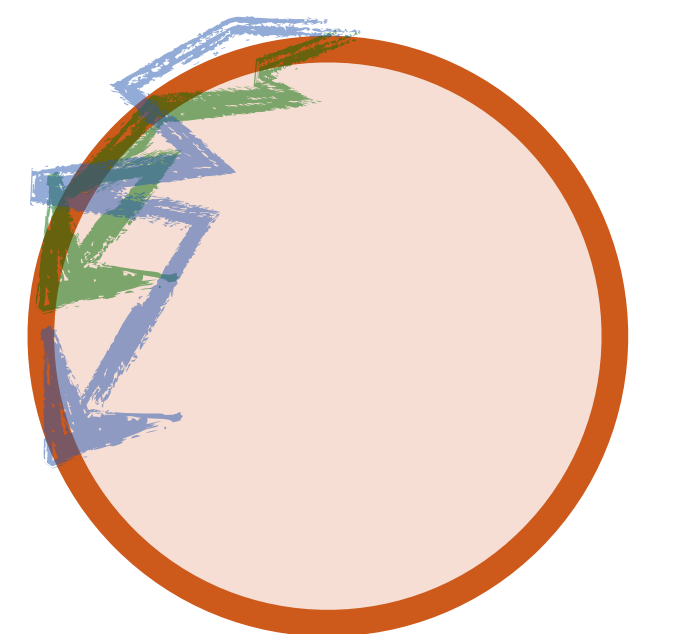
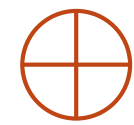
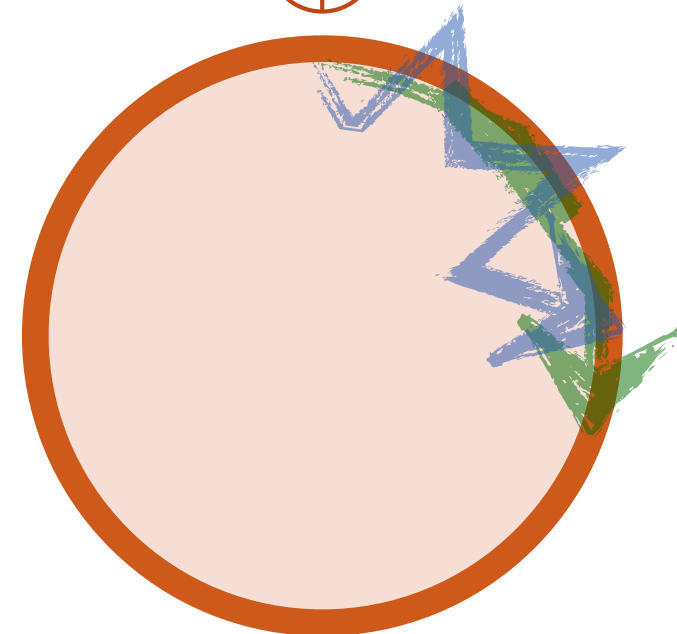
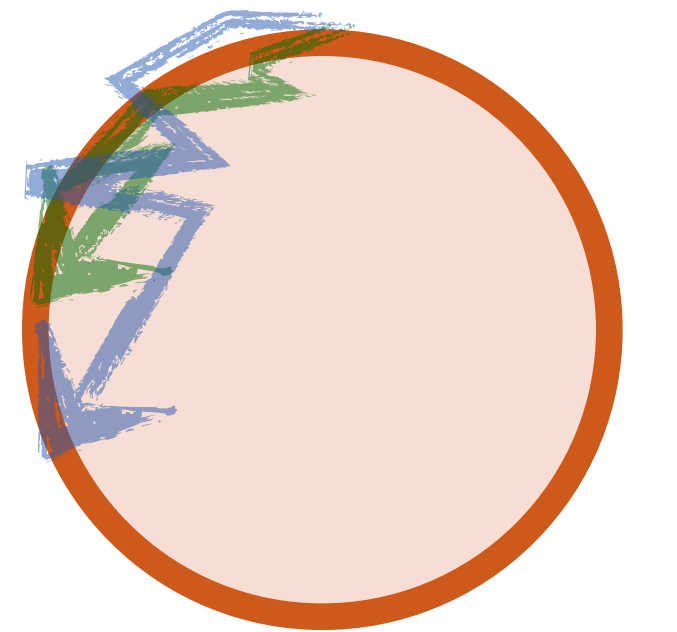


Data randomness leads to decoherence for generic rotations  $V_{z_M} \cdots V_{z_1}$

**✗ no quantum advantage**

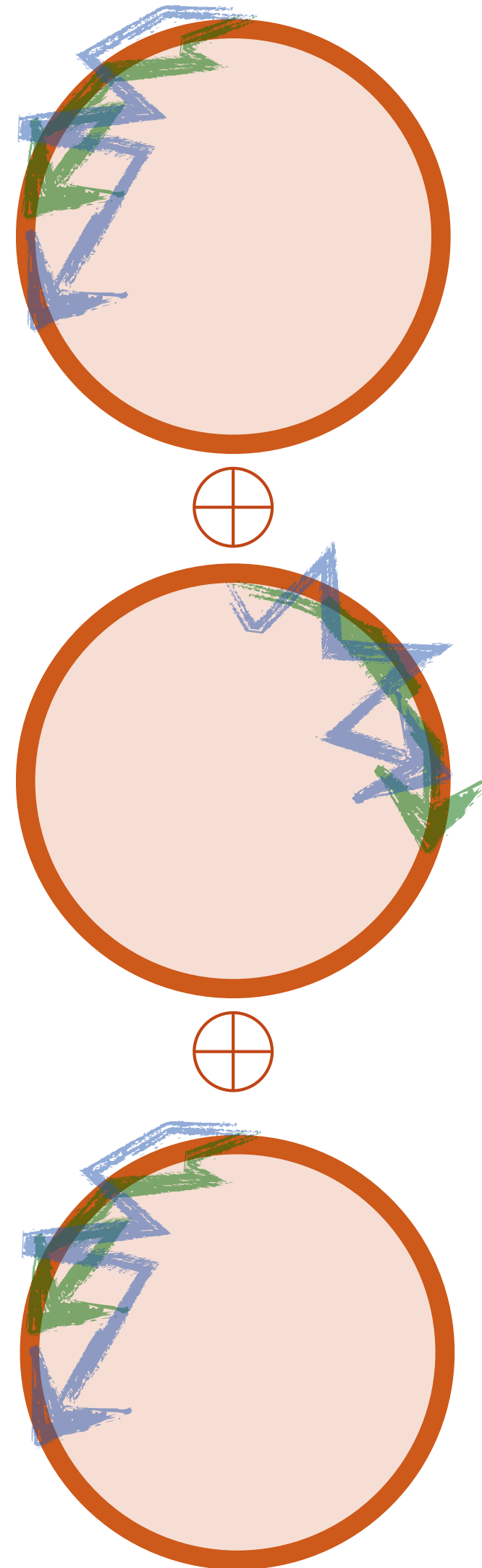
[randomized Hamiltonian simulation; qDrift]

# Quantum oracle sketching



Carefully design the rotations  
to be nearly orthogonal  
to avoid decoherence.

# Quantum oracle sketching



Carefully design the rotations to be nearly orthogonal to avoid decoherence.

## Theorem (Quantum Oracle Sketch)

With  $\Theta(NQ^2)$  samples, we can emulate  $Q$  quantum queries to an oracle of  $N$  items with no memory overhead.

# Toy example

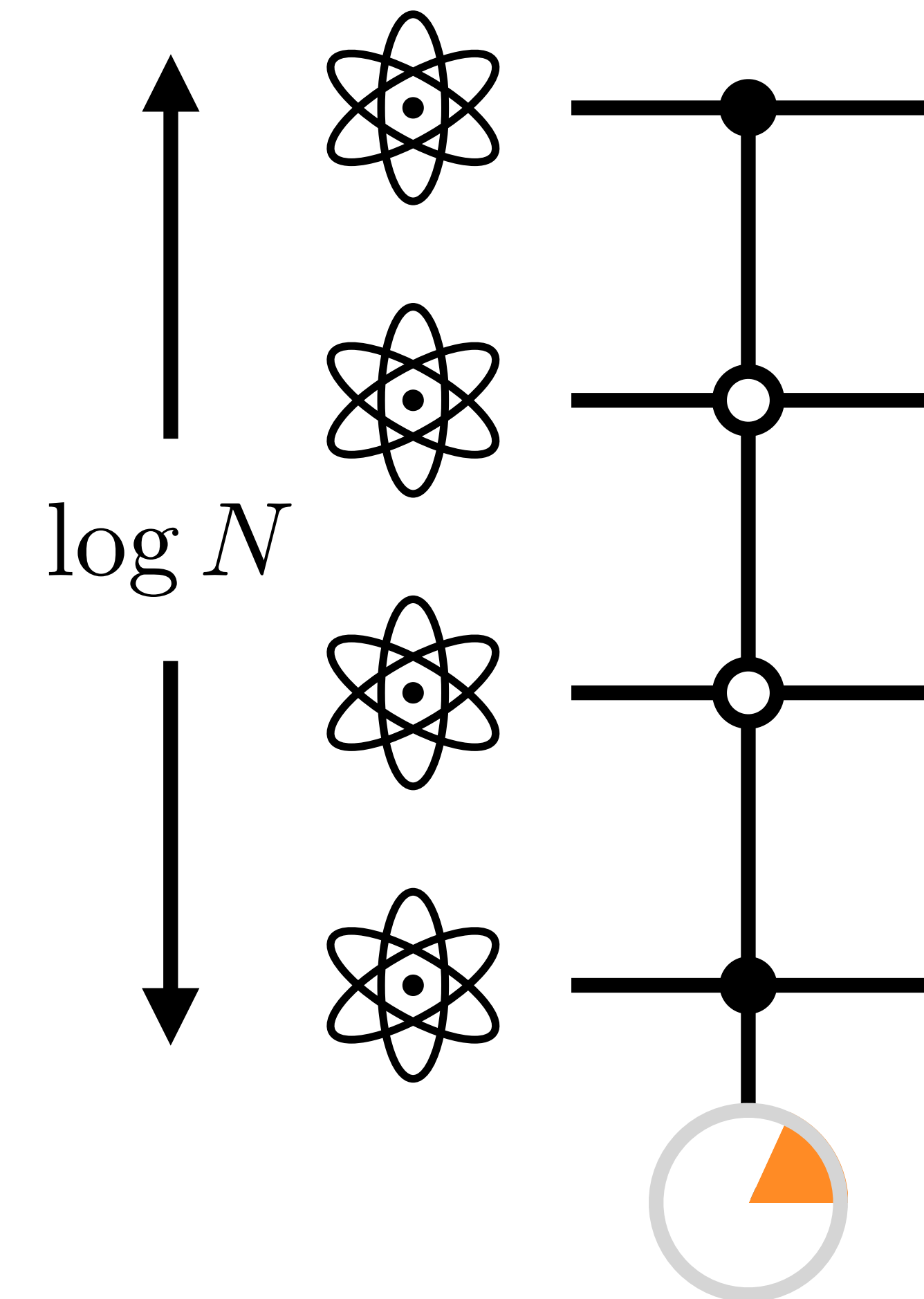
Example

Goal: estimate property of  $f : [N] \rightarrow \{0, 1\}$ , given  $\{x_i, f(x_i)\}_{i=1}^M, p(x) = 1/N$

# Toy example

Example

Goal: estimate property of  $f : [N] \rightarrow \{0, 1\}$ , given  $\{x_i, f(x_i)\}_{i=1}^M, p(x) = 1/N$



$$x_i = 1001$$

$$f(x_i) = 1$$

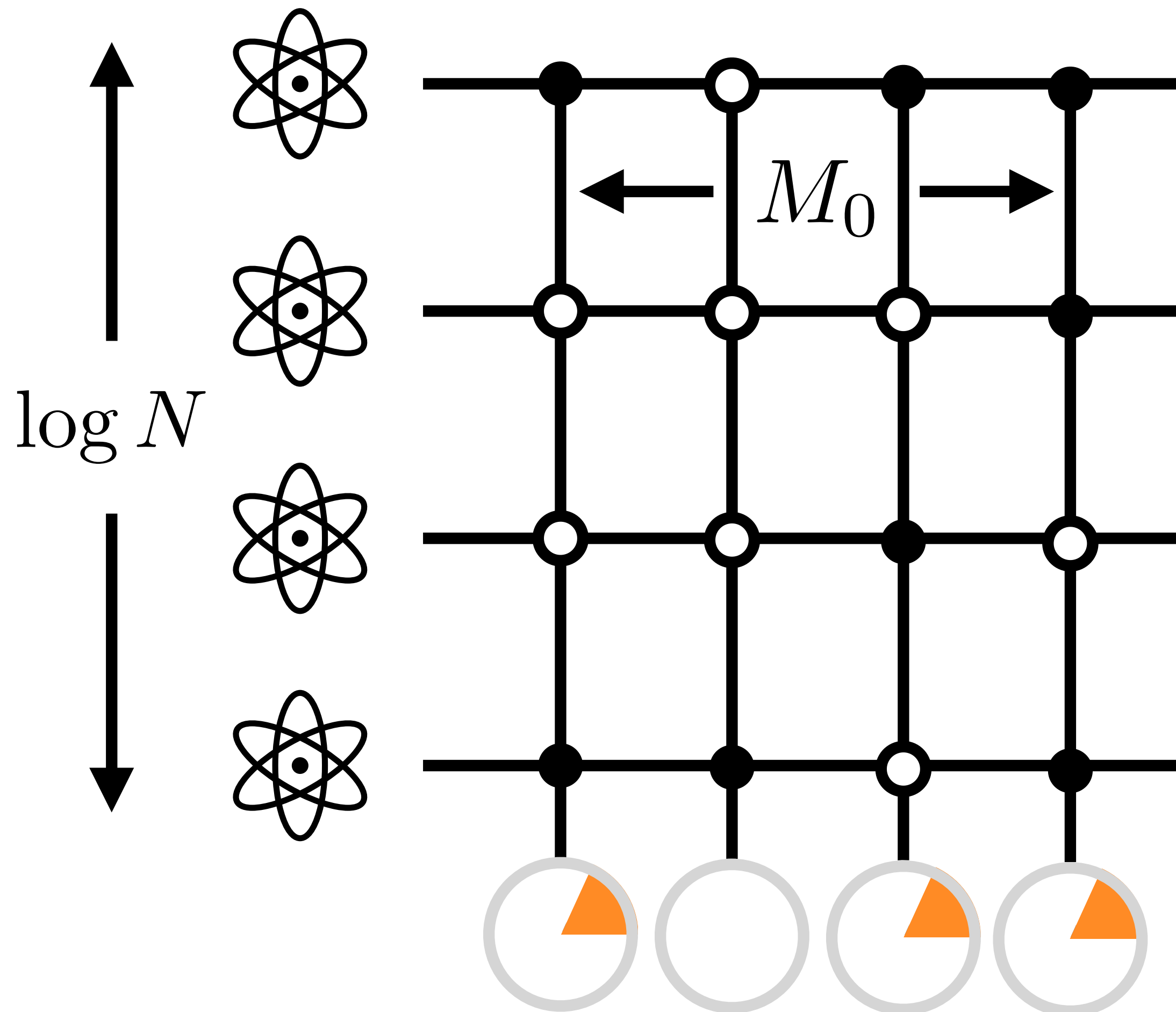
**multi-controlled phase gate**

$$V_i = \exp \left( i \frac{t}{M_0} f(x_i) |x_i\rangle \langle x_i| \right)$$

# Toy example

Example

Goal: estimate property of  $f : [N] \rightarrow \{0, 1\}$ , given  $\{x_i, f(x_i)\}_{i=1}^M, p(x) = 1/N$



$$V_i = \exp \left( i \frac{t}{M_0} f(x_i) |x_i\rangle \langle x_i| \right)$$

$$V_{M_0} \cdots V_1$$

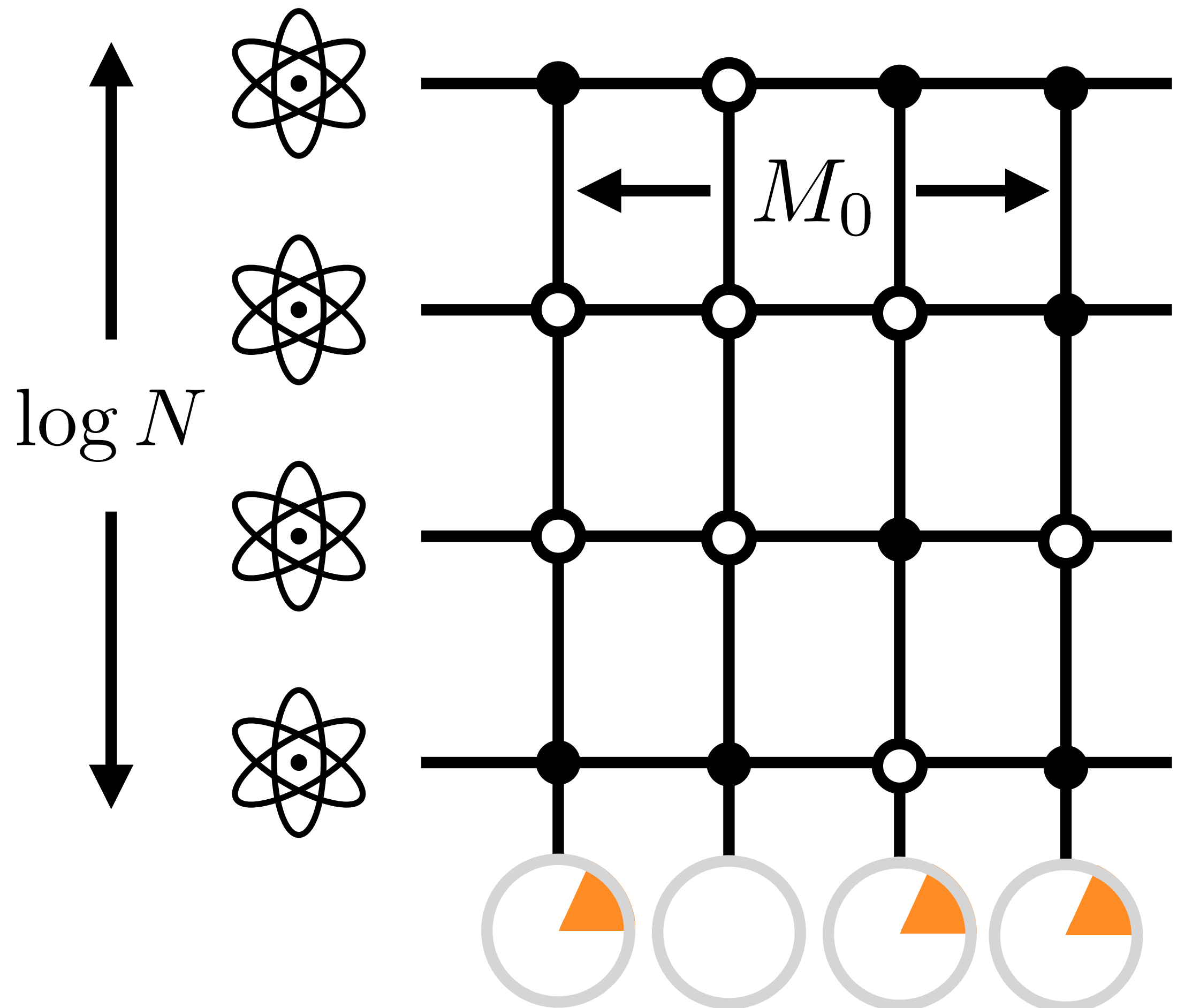
$$= \exp \left( it \sum_x m_x f(x) |x\rangle \langle x| \right)$$

frequency of  $x_i = x$

# Toy example

Example

Goal: estimate property of  $f : [N] \rightarrow \{0, 1\}$ , given  $\{x_i, f(x_i)\}_{i=1}^M, p(x) = 1/N$



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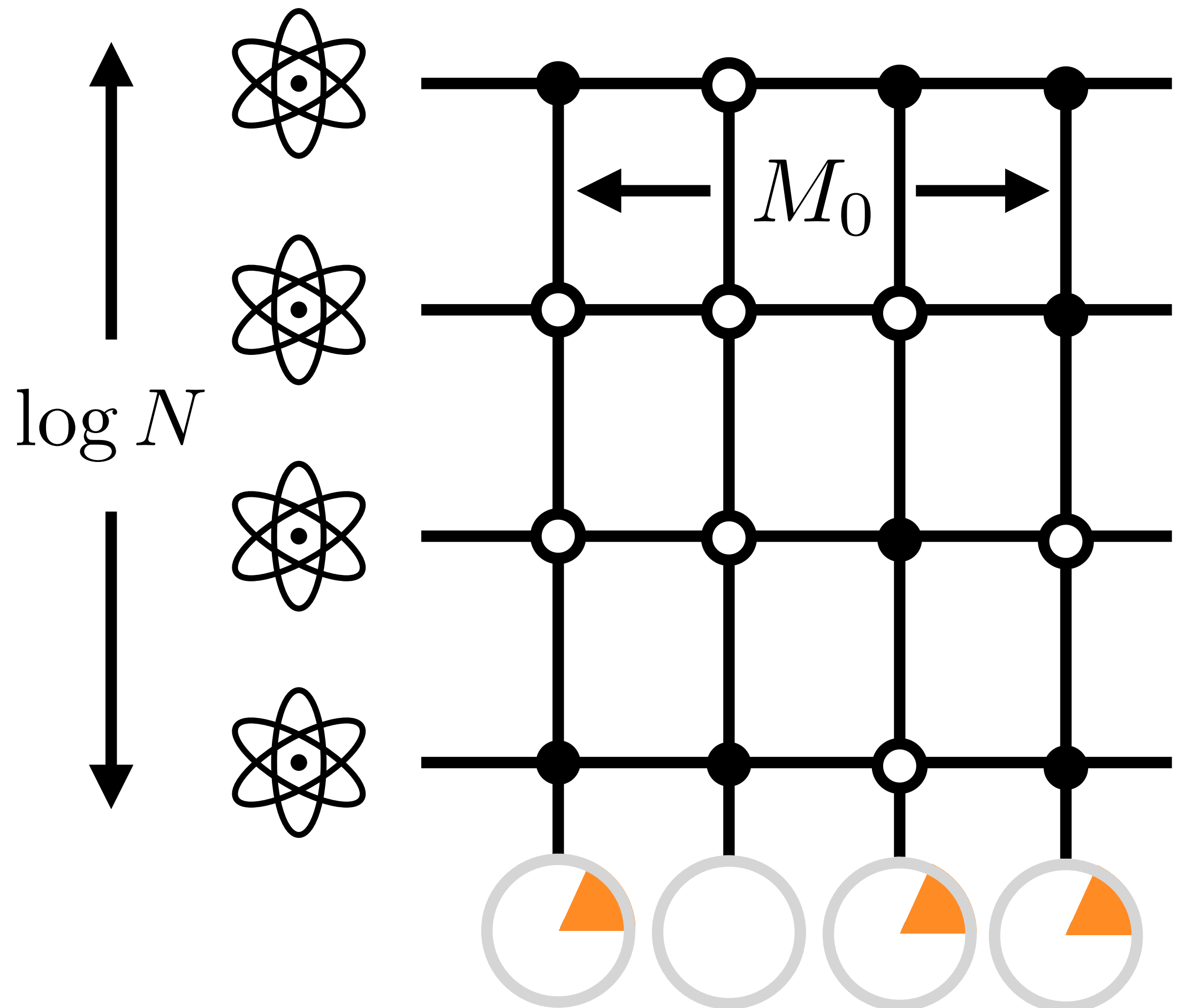
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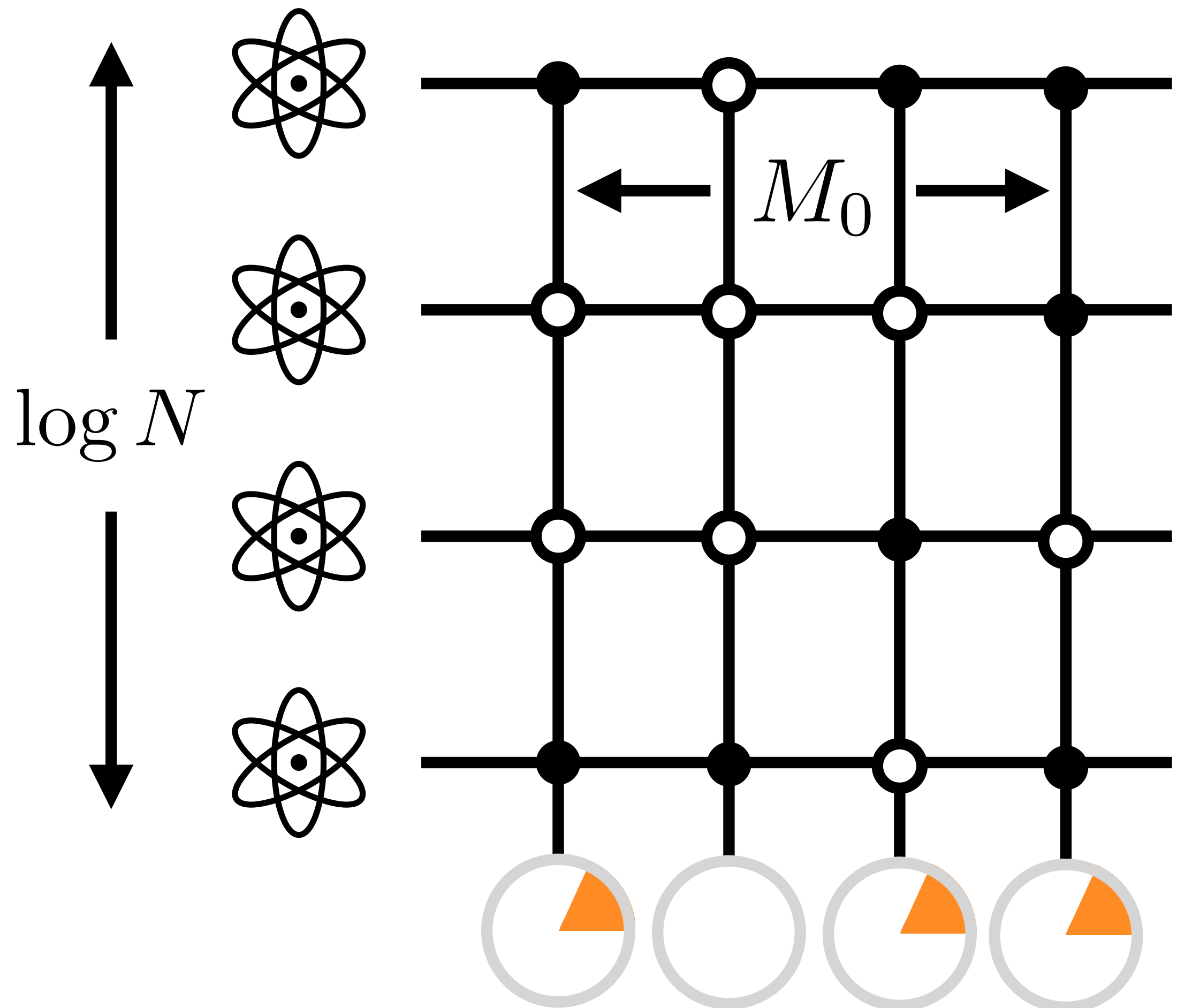
$t = \pi N$        $m_x \approx p(x) = 1/N$

$$\approx \sum_x \exp(i\pi f(x)) |x\rangle \langle x|$$

# Toy example

Example

Goal: estimate property of  $f : [N] \rightarrow \{0, 1\}$ , given  $\{x_i, f(x_i)\}_{i=1}^M, p(x) = 1/N$

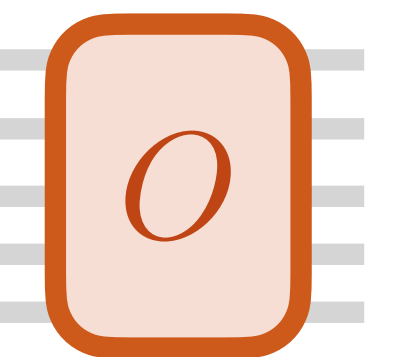


$$V_i = \exp \left( i \frac{t}{M_0} f(x_i) |x_i\rangle \langle x_i| \right)$$

$$V_{M_0} \cdots V_1$$

$$\approx_{\epsilon} \sum_x (-1)^{f(x)} |x\rangle \langle x|$$

phase oracle



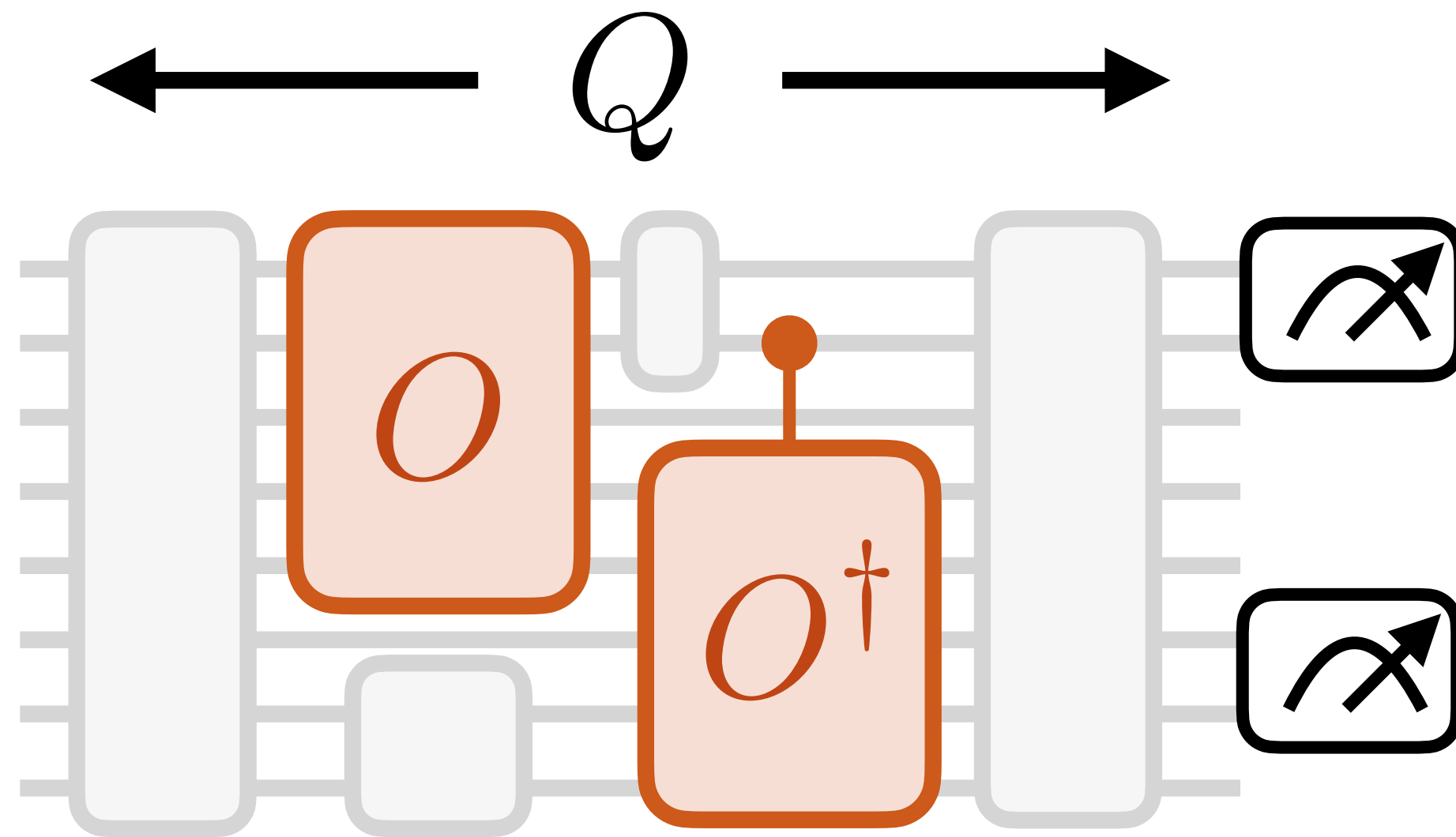
Theorem

$$M_0 = \Theta(N/\epsilon)$$

# Toy example

Example

Goal: estimate property of  $f : [N] \rightarrow \{0, 1\}$ , given  $\{x_i, f(x_i)\}_{i=1}^M, p(x) = 1/N$



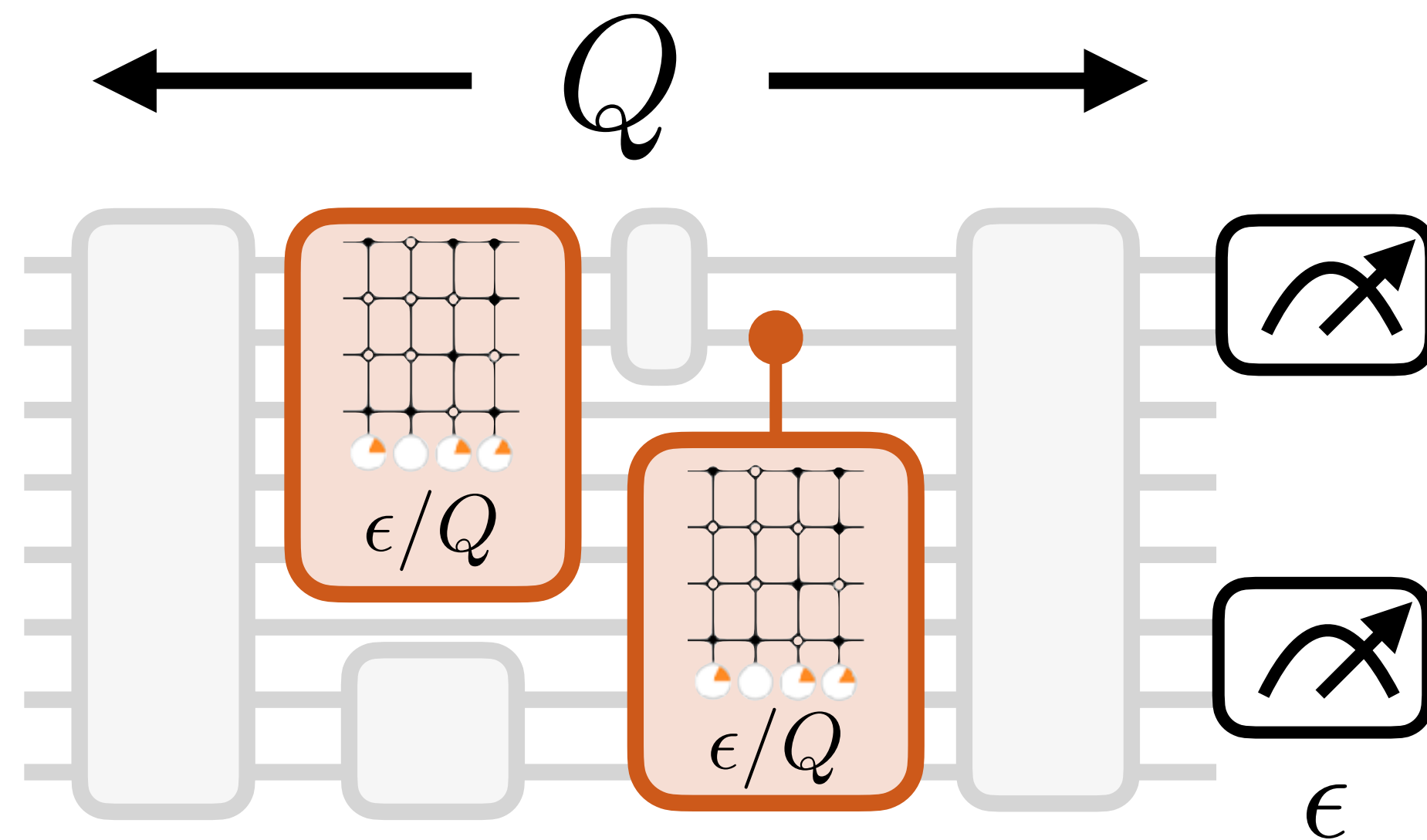
any quantum query algorithm

making  $Q$  queries

# Toy example

Example

Goal: estimate property of  $f : [N] \rightarrow \{0, 1\}$ , given  $\{x_i, f(x_i)\}_{i=1}^M, p(x) = 1/N$



any quantum query algorithm

making  $Q$  queries

$$M = Q \cdot \underbrace{O\left(\frac{N}{\epsilon/Q}\right)}_{M_0} = O\left(\frac{NQ^2}{\epsilon}\right)$$

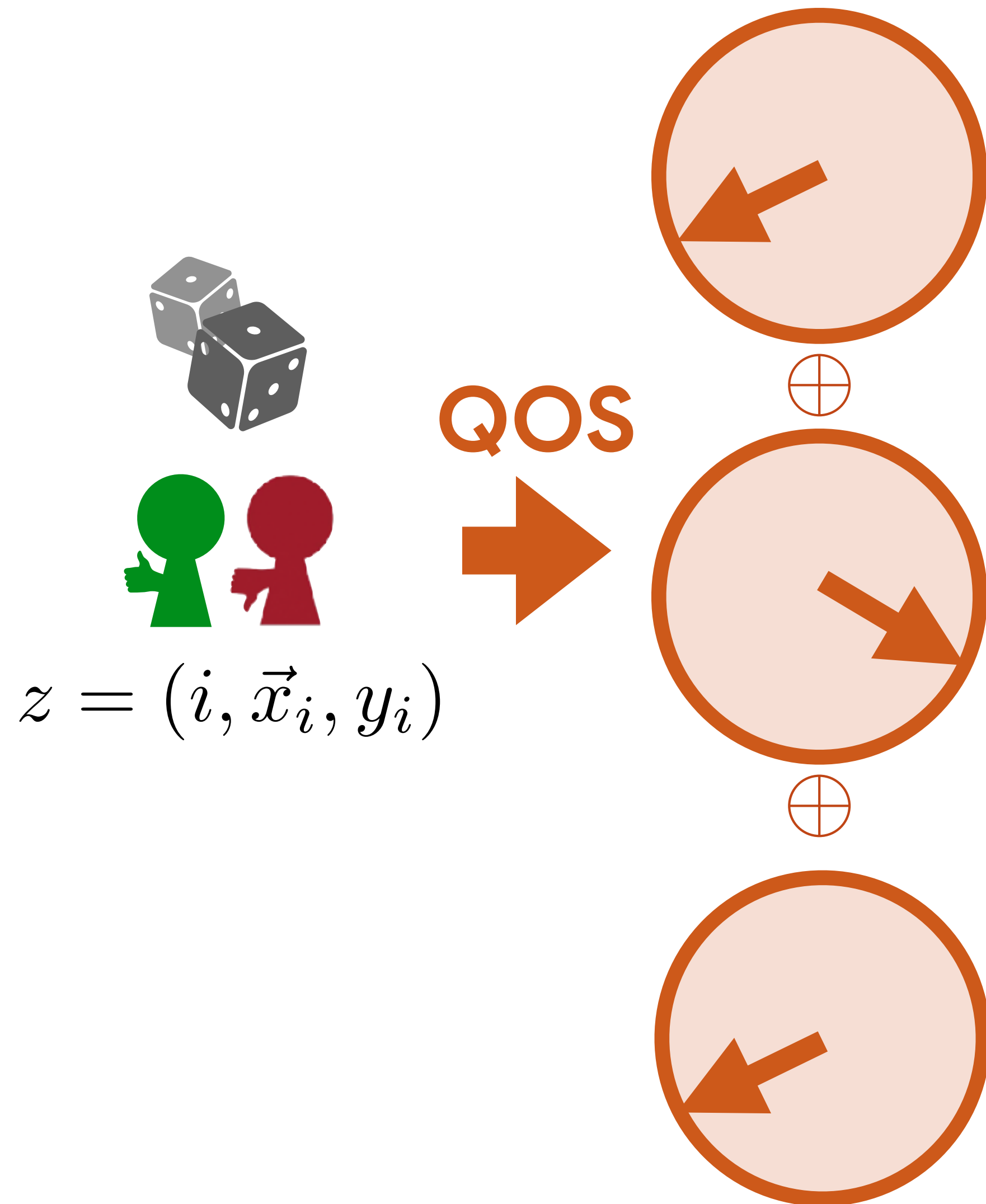
# Quantum oracle sketching

Quadratic slowdown in  $Q$  is the fundamental price to pay.


incoherent sampling of data,  $\text{amp}^2 = \text{prob}$ ,  $\text{HL}^2 = \text{SQL}$

Theorem (Quantum Oracle Sketch)

With  $\Theta(NQ^2)$  samples, we can emulate  $Q$  quantum queries to an oracle of  $N$  items with no memory overhead.



# Quantum oracle sketching

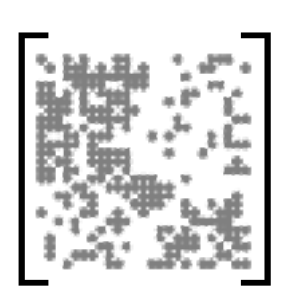

$$z = (x, f(x))$$

$$|x\rangle \rightarrow (-1)^{f(x)} |x\rangle$$

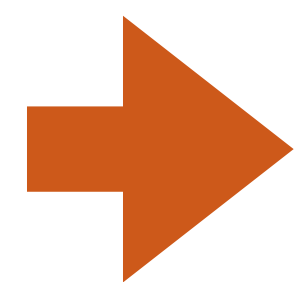
phase oracle

$$|x\rangle |y\rangle \rightarrow |x\rangle |y \oplus f(x)\rangle$$

XOR oracle

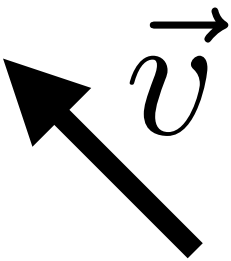

$$z = (i, j, A_{ij})$$

**QOS**



$$U = \begin{pmatrix} A & \star \\ \star & \star \end{pmatrix}$$

block encoding


$$z = (k, v_k)$$

$$|0\rangle \rightarrow |v\rangle$$

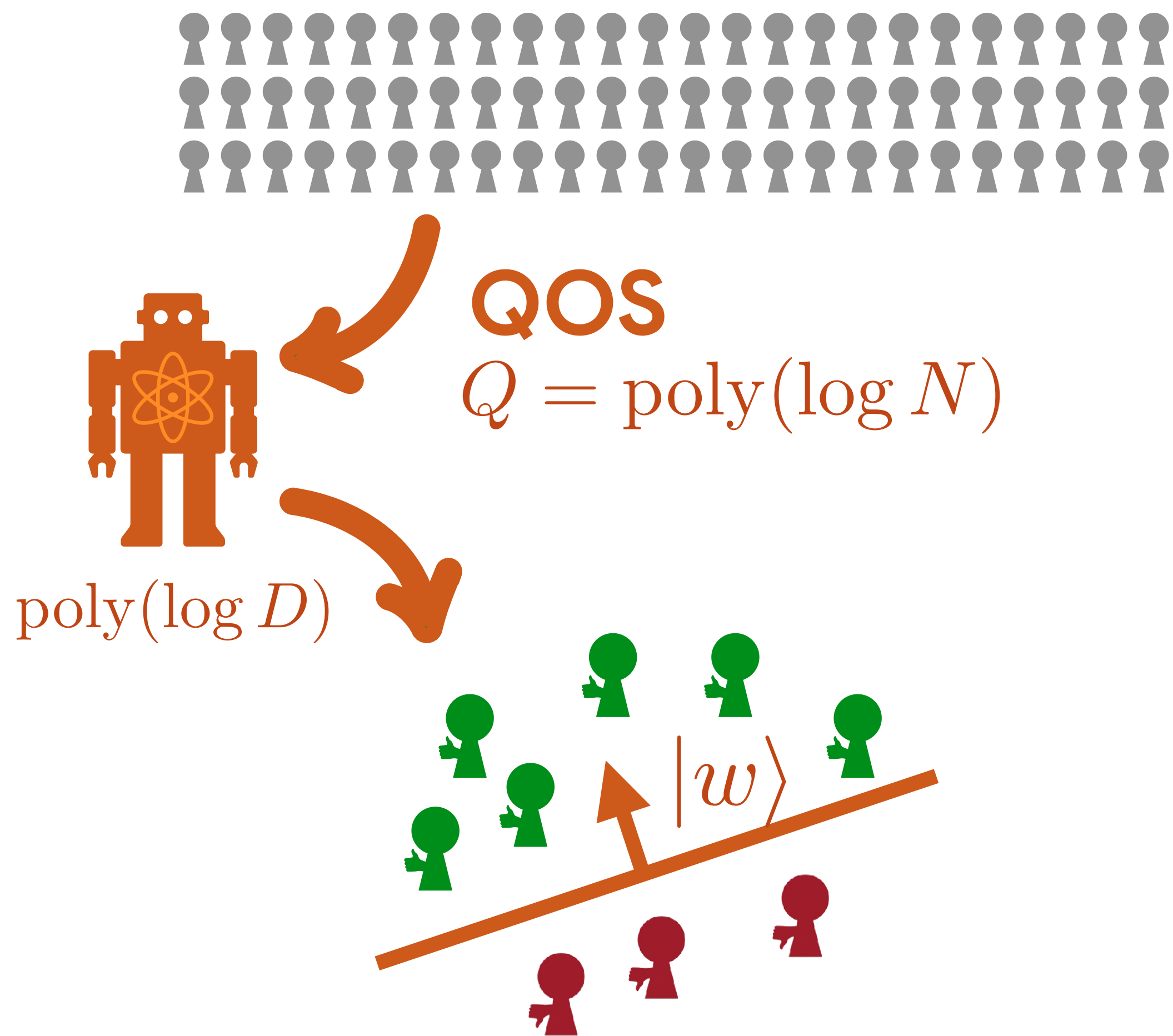
state preparation  
unitary

**Robust to noise and correlation in data; support flexible data structures.**

Theorem (Quantum Oracle Sketch)

With  $\Theta(NQ^2)$  samples, we can emulate  $Q$  quantum queries to an oracle of  $N$  items with no memory overhead.

# Quantum oracle sketching



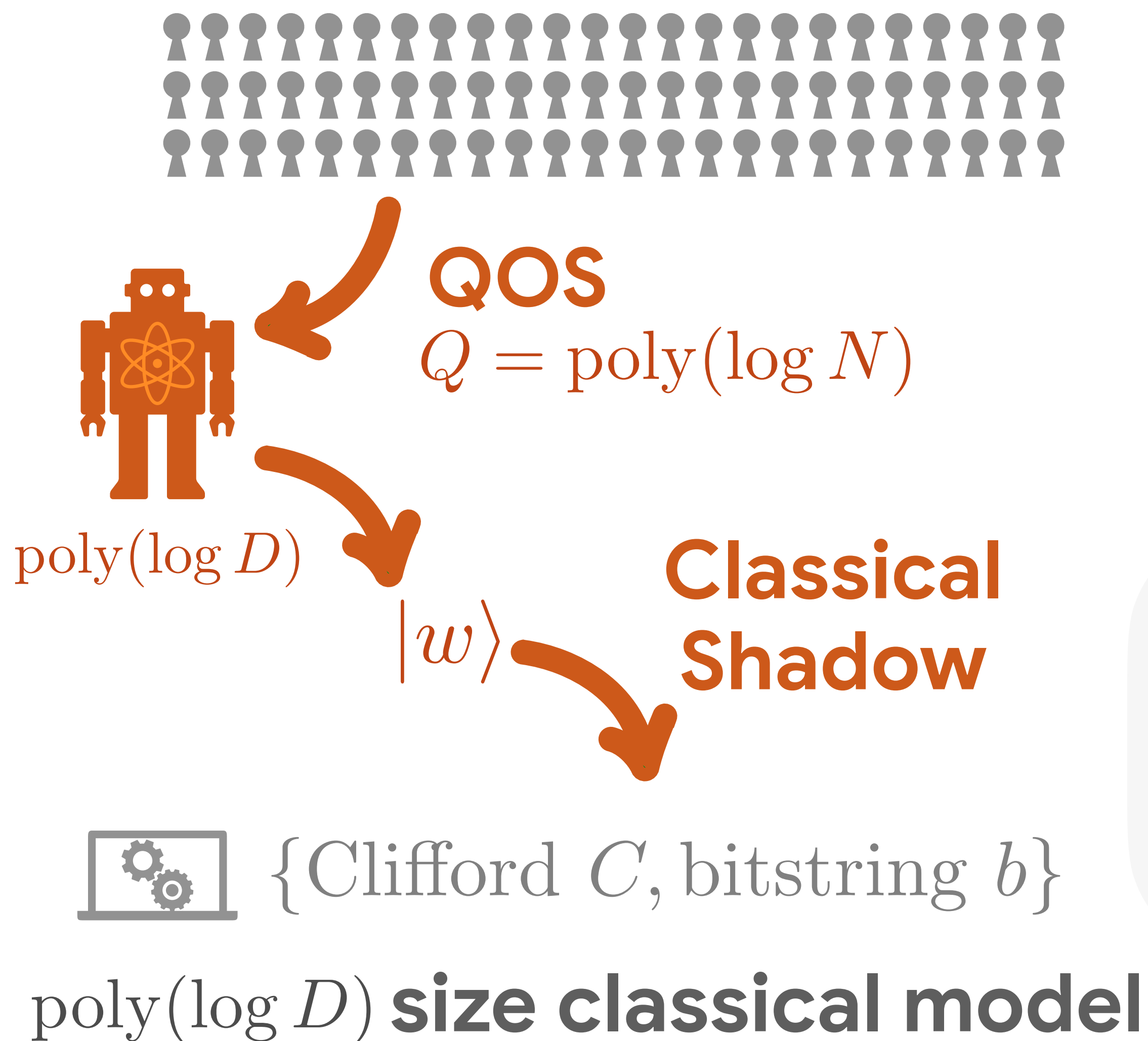
**ML tasks can be solved with  $\text{poly}(\log D)$  size and  $Q = \text{poly}(\log N)$  queries.**

SVM: linear system  
PCA: ground state prep

## Theorem (Quantum Oracle Sketch)

With  $\tilde{\Theta}(N)$  samples, a quantum machine of  $\text{poly}(\log D)$  size can solve the ML tasks.

# Quantum oracle sketching

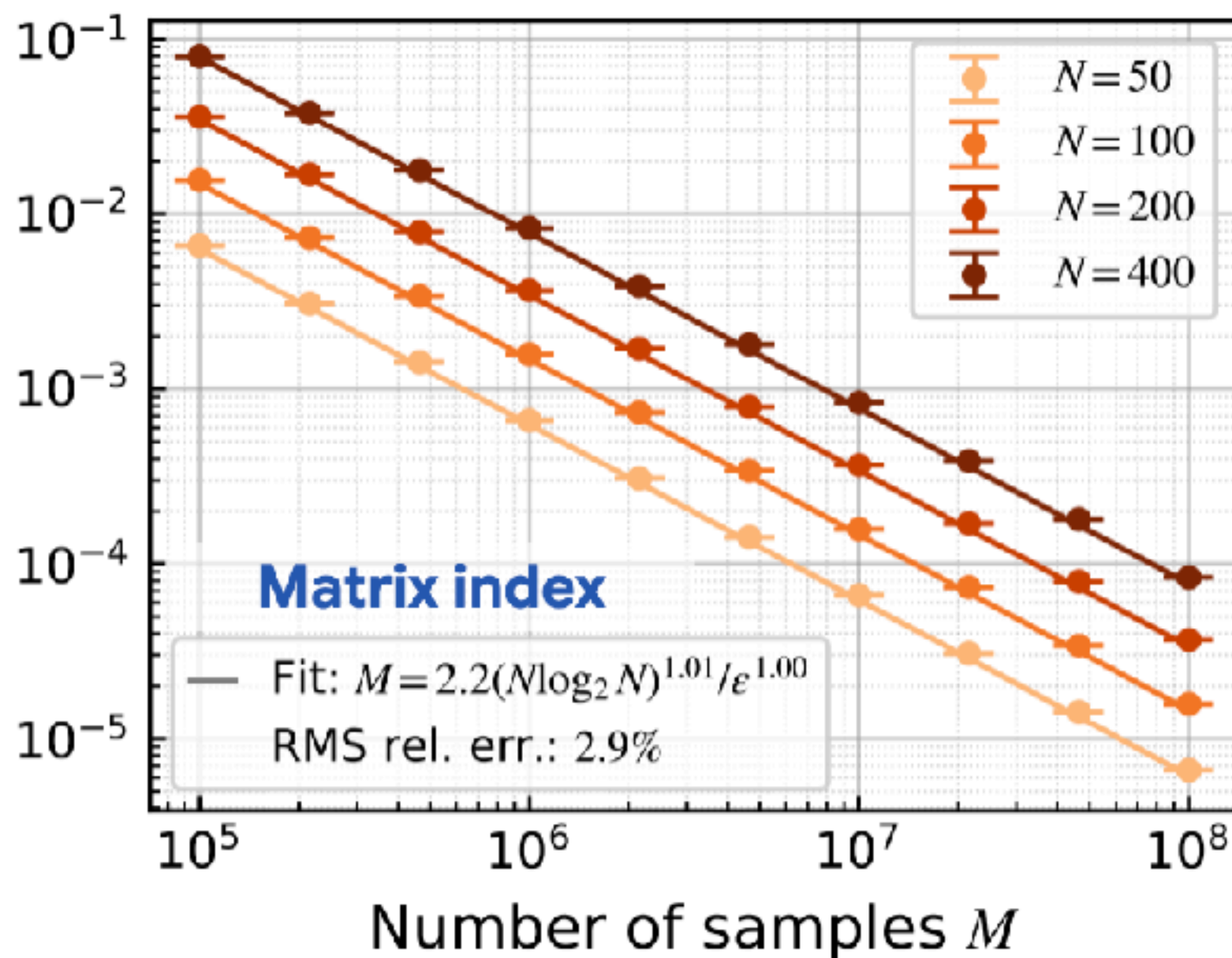
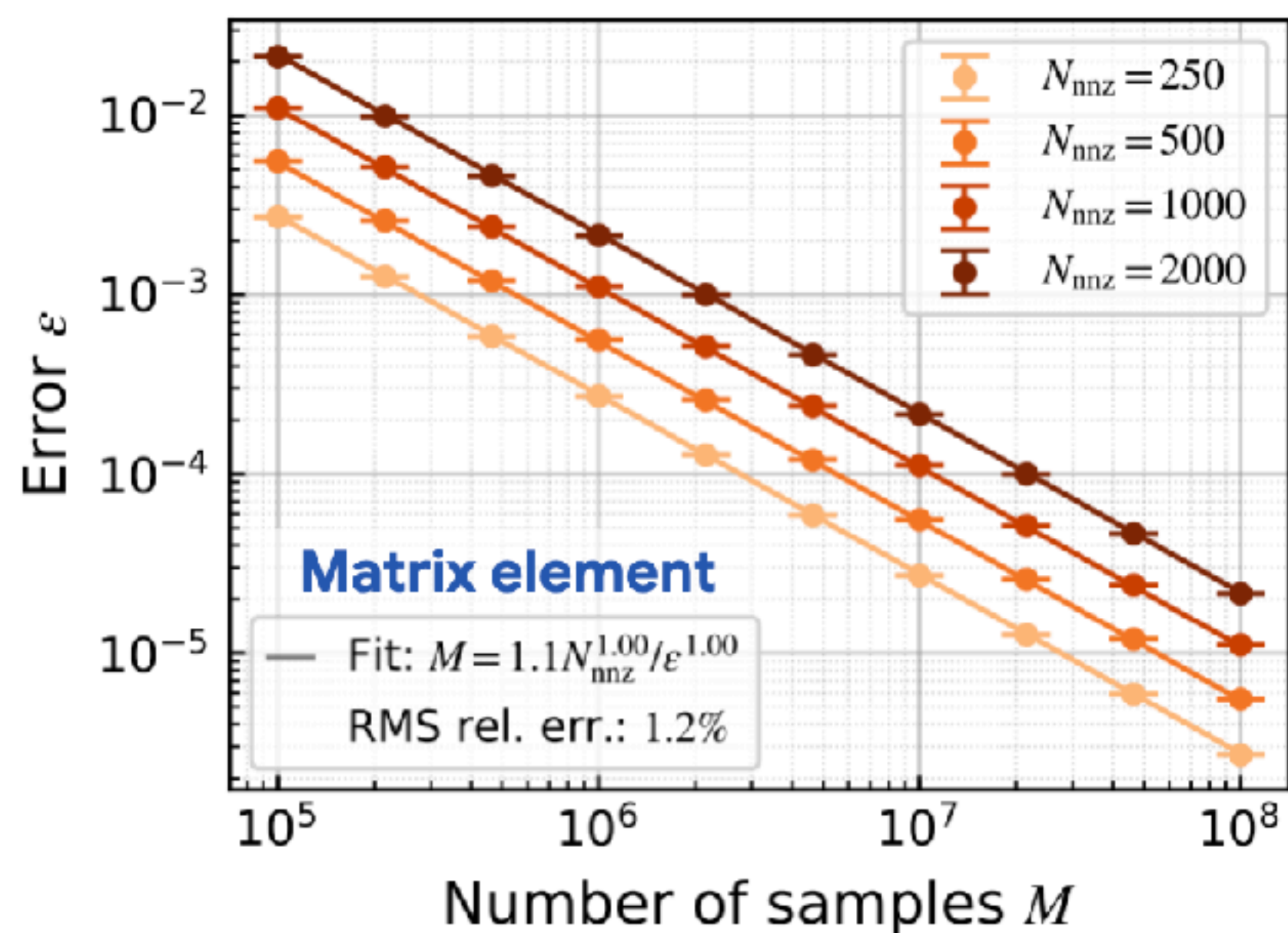
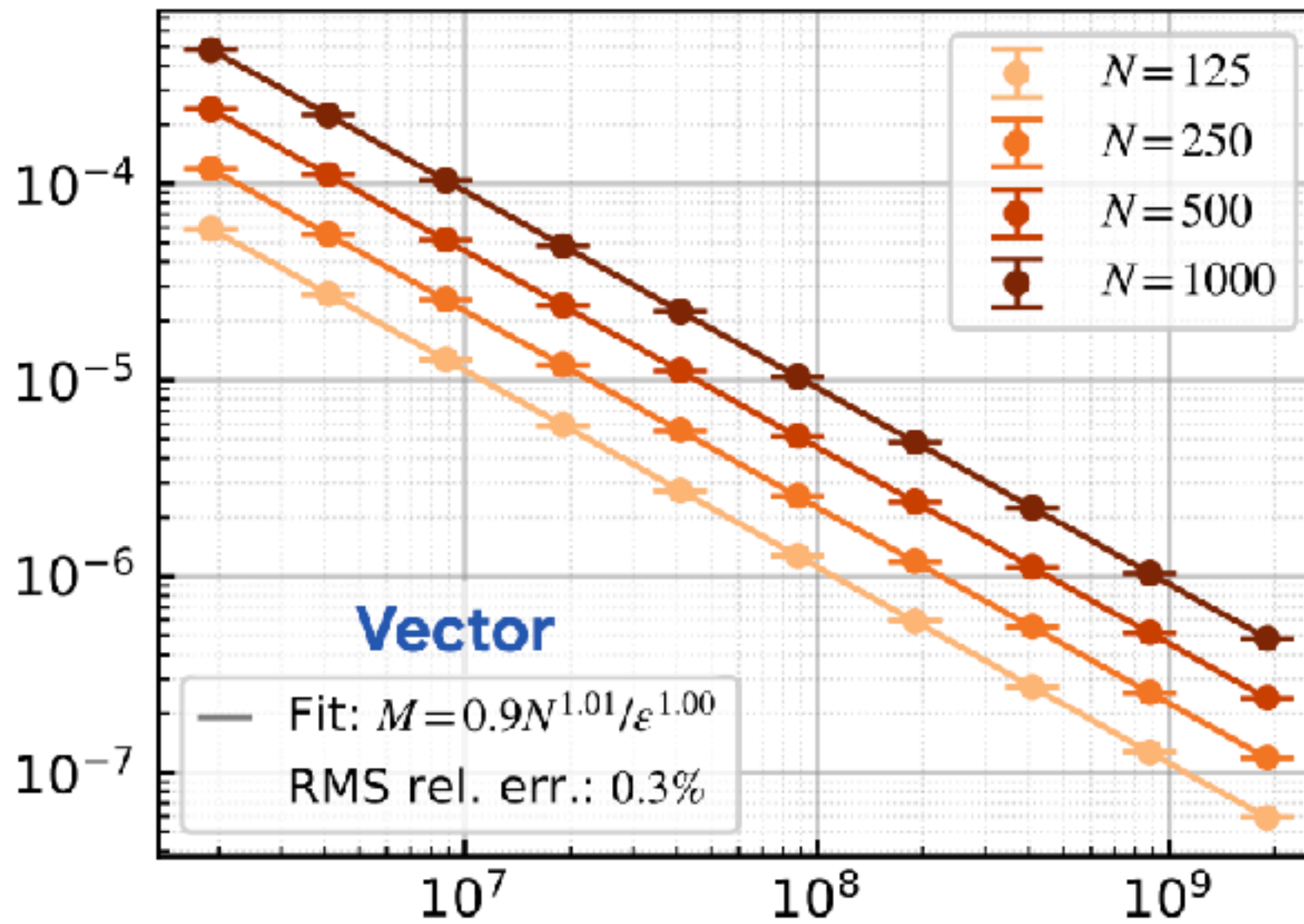
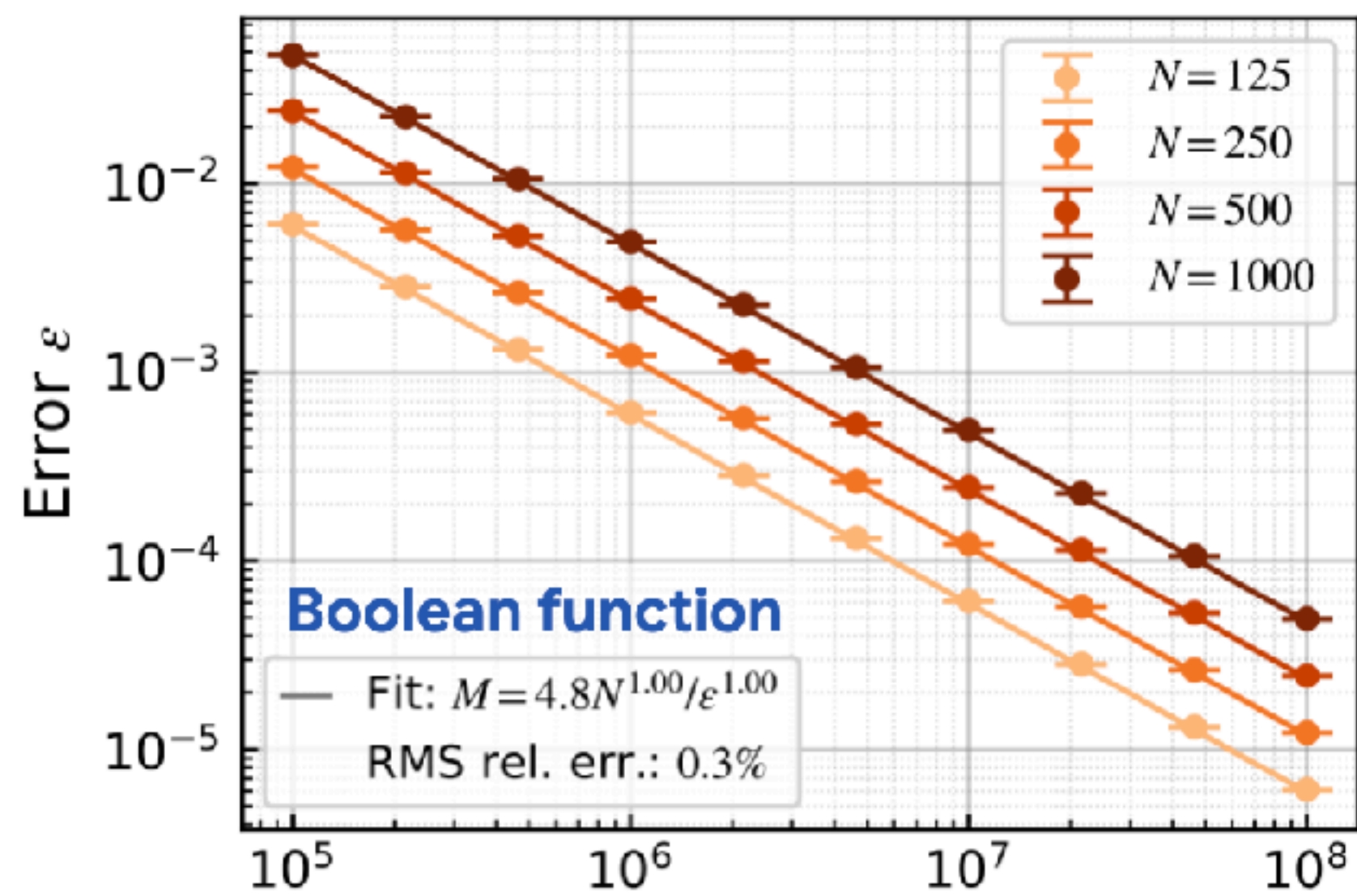


**Interferometric classical shadow:**  
exponentially compact classical model  
possible only via quantum technology.

## Theorem (Quantum Oracle Sketch)

With  $\tilde{\Theta}(N)$  samples, a quantum machine of  $\text{poly}(\log D)$  size can solve the ML tasks.

# Quantum oracle sketching



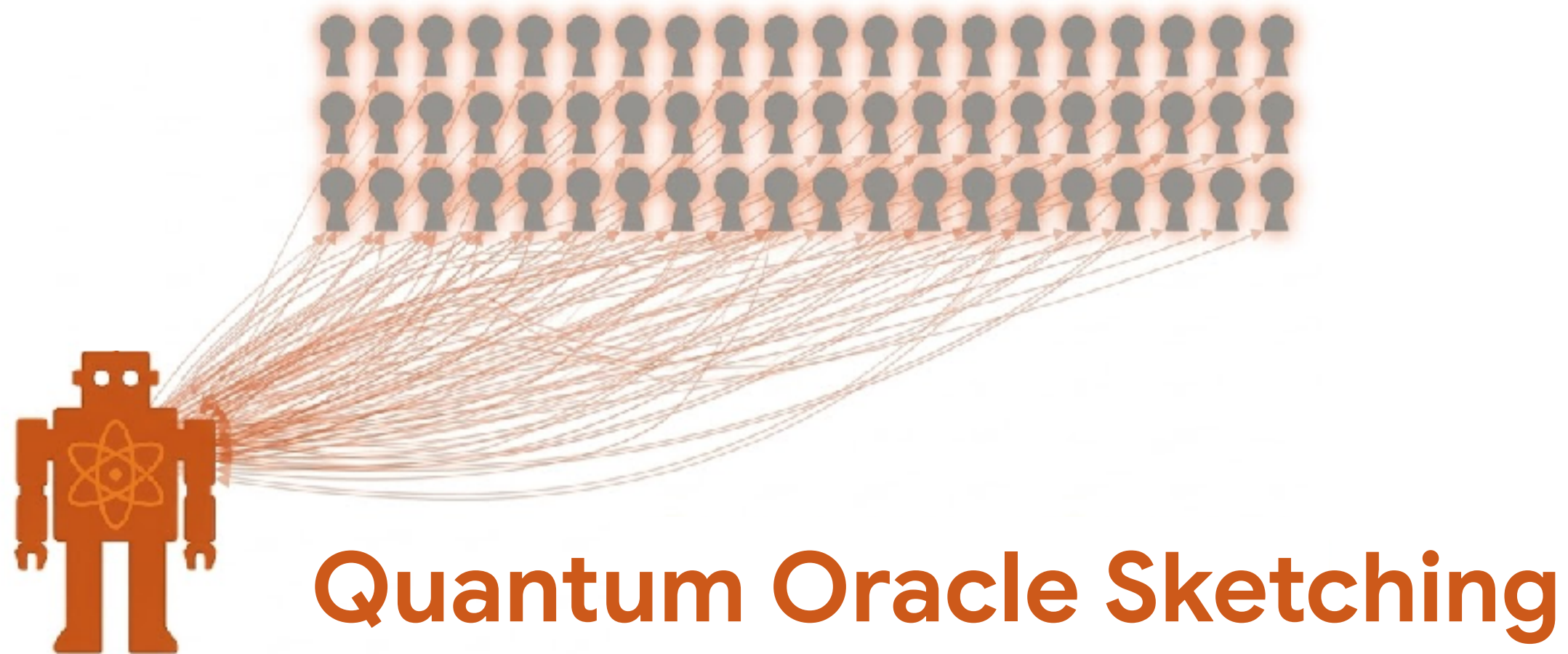
GitHub

haimengzhao/  
quantum-oracle-  
sketching



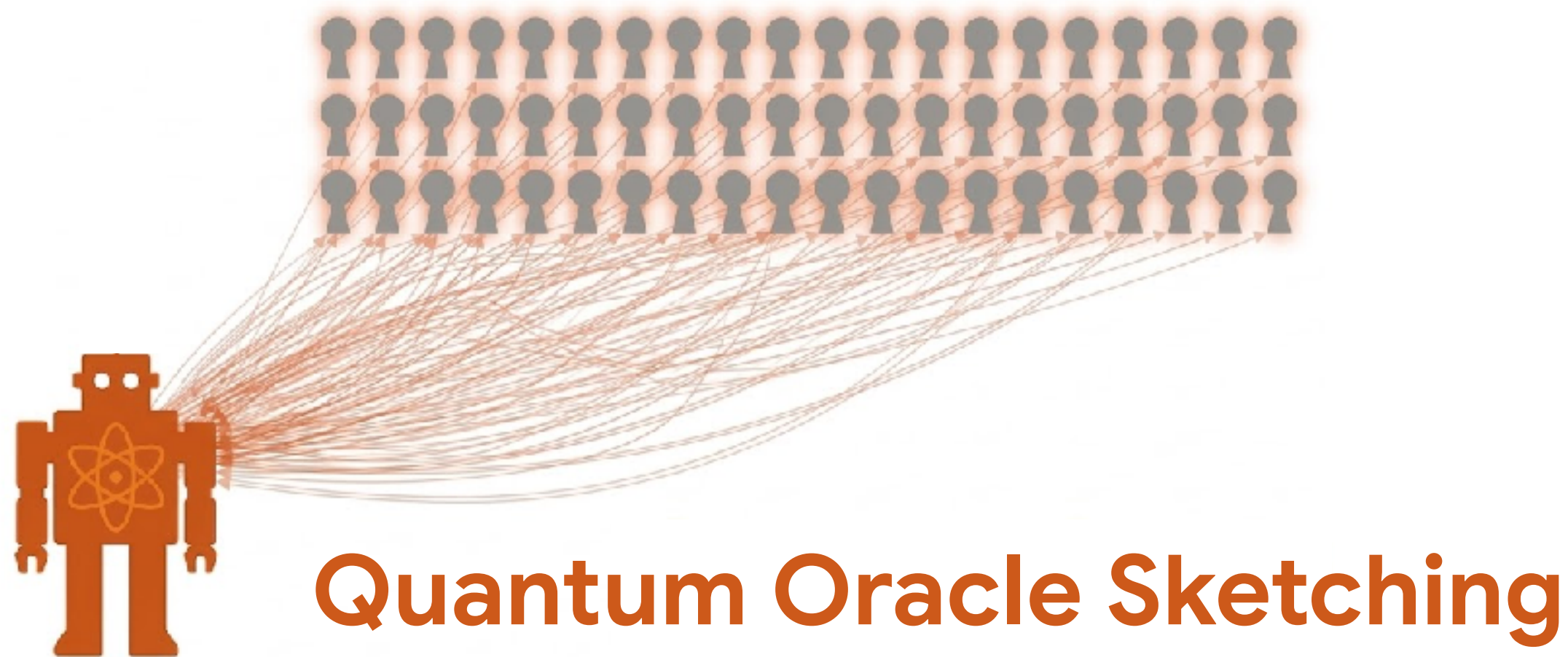
JAX-powered,  
support GPU/TPU,  
auto-differentiation

# Quantum Algorithm



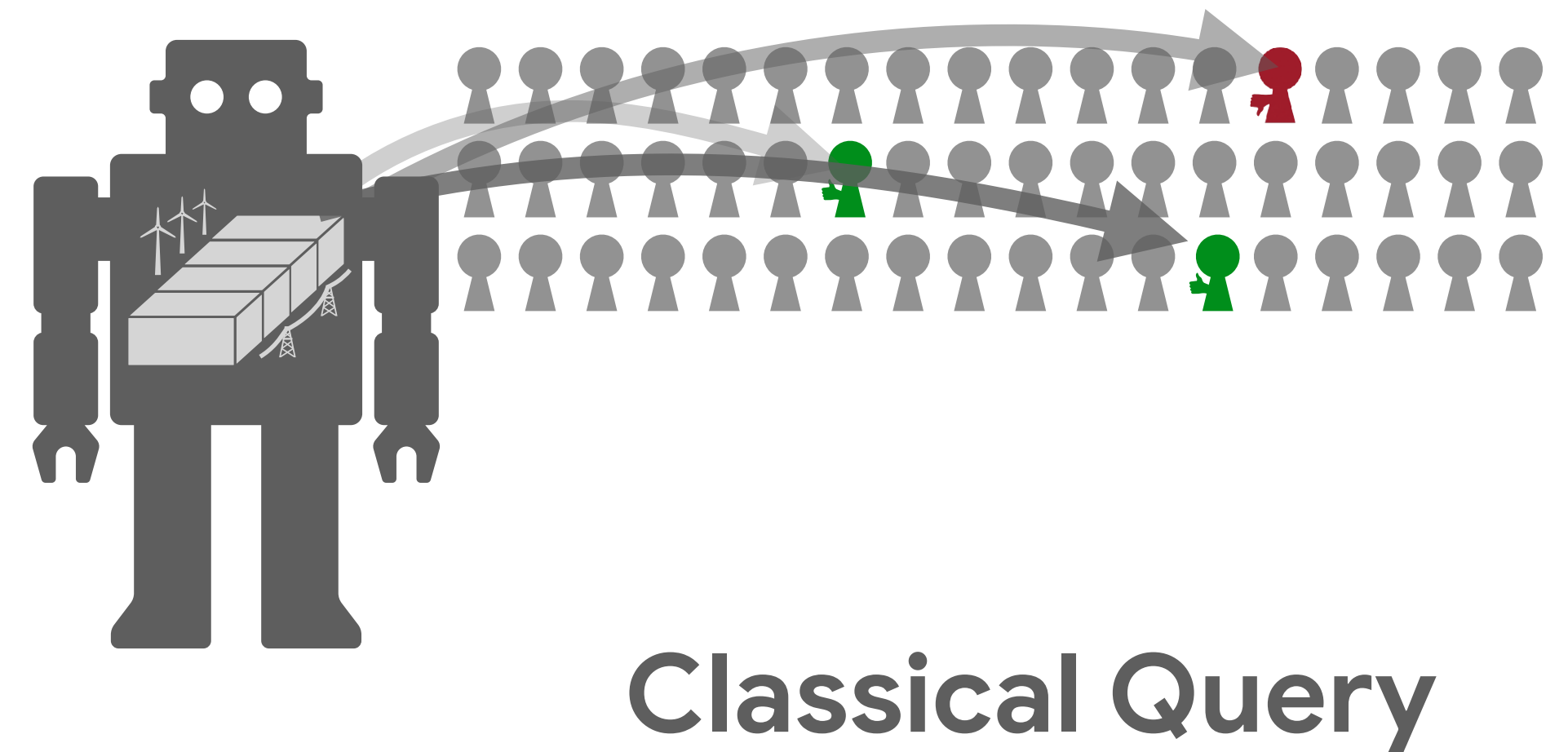
**query the classical world in  
quantum superposition**

## Quantum Algorithm



query the classical world in  
quantum superposition

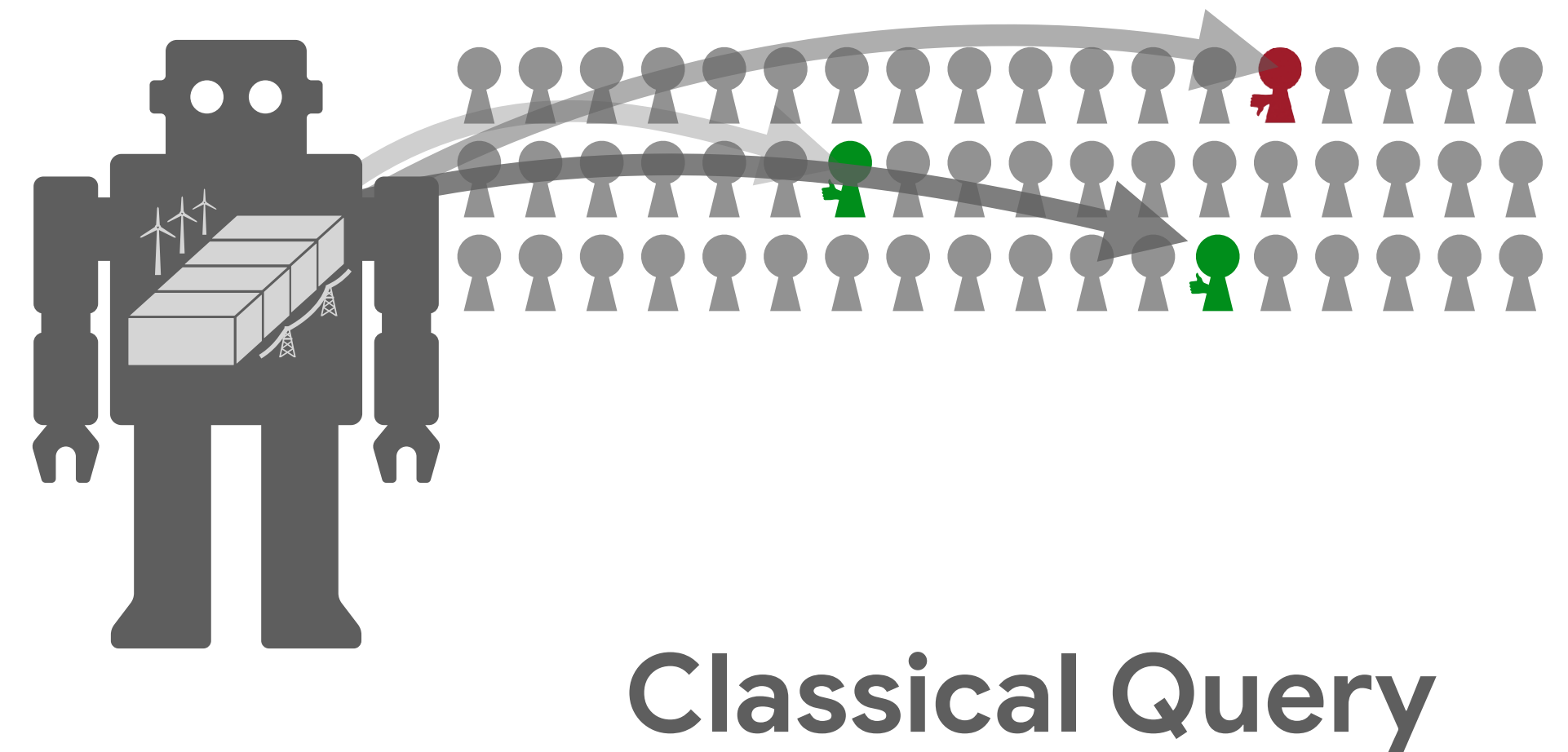
## Classical Hardness



separation in query ability  
→ memory advantage

# Classical Hardness

1. Connect query separation to memory advantage
2. Insufficient memory must be compensated by more samples
3. Embed into ML tasks



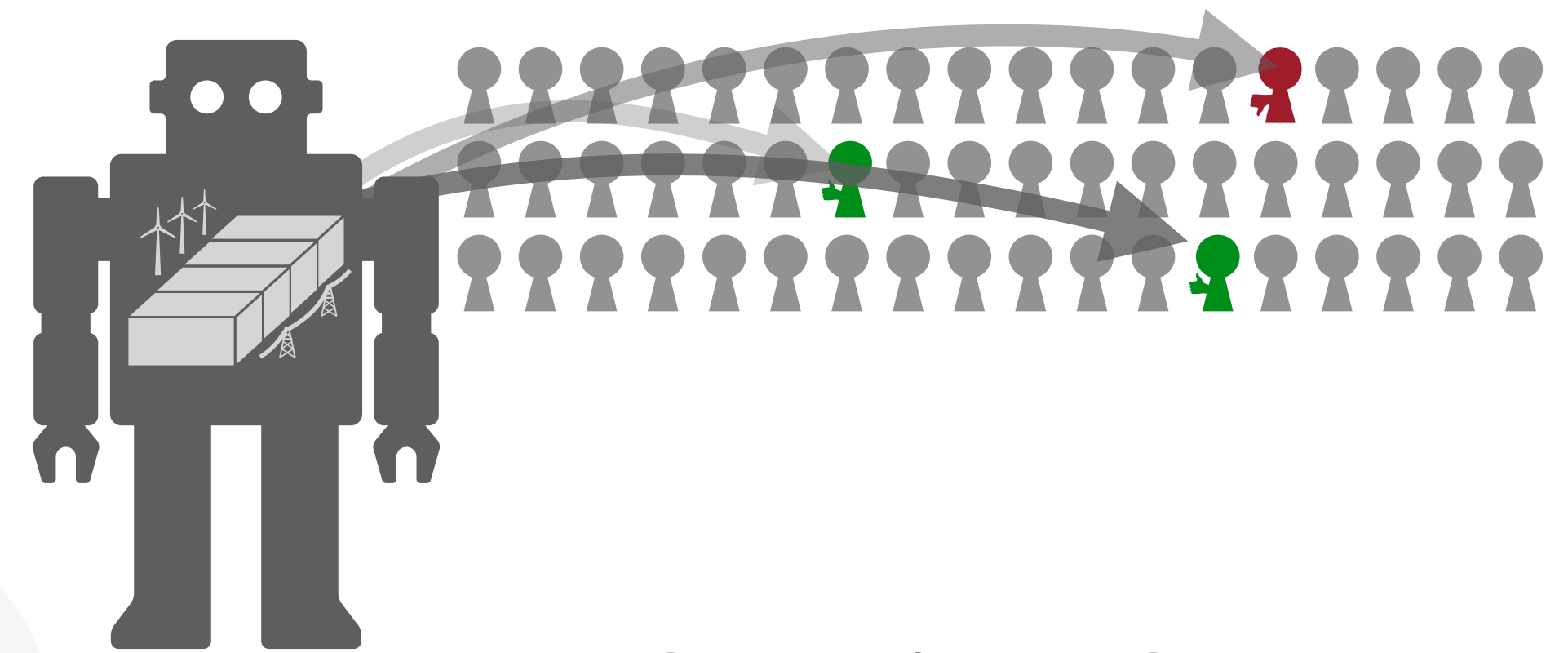
# Classical Hardness

## 1. Connect query separation to memory advantage

### Theorem (Classical Hardness)

For any query problem requiring  $Q$  quantum queries or  $Q_C$  classical queries, to solve an associated learning task with  $\Theta(NQ^2)$  samples, any classical machine must have size

$$S \geq \Omega(Q_C/Q^2)$$



Classical Query

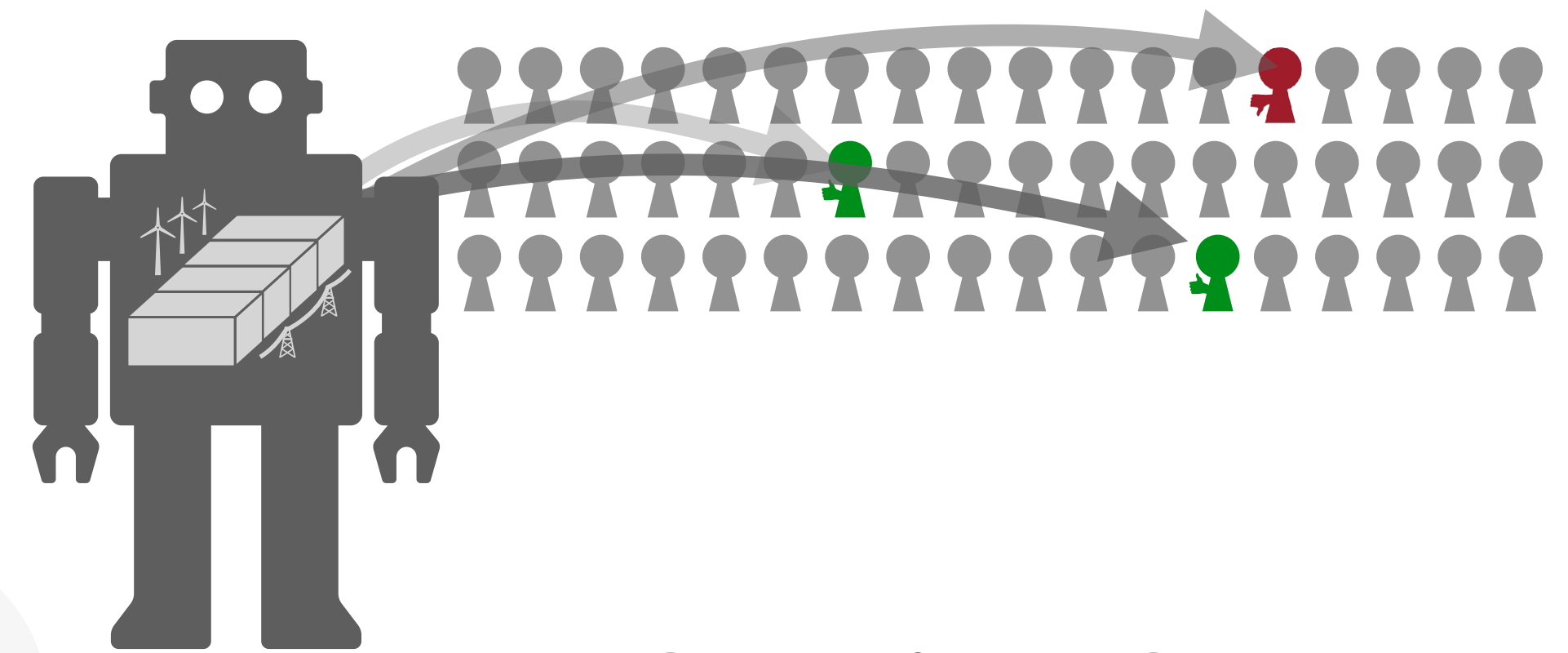
# Classical Hardness

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$$S \geq \Omega(Q_C/Q^2)$$



Classical Query

Any super-quadratic query separation,

$$Q = O(N^{0.49}), Q_C = \Theta(N)$$

$$\implies S \geq \Omega(N^{0.02})$$

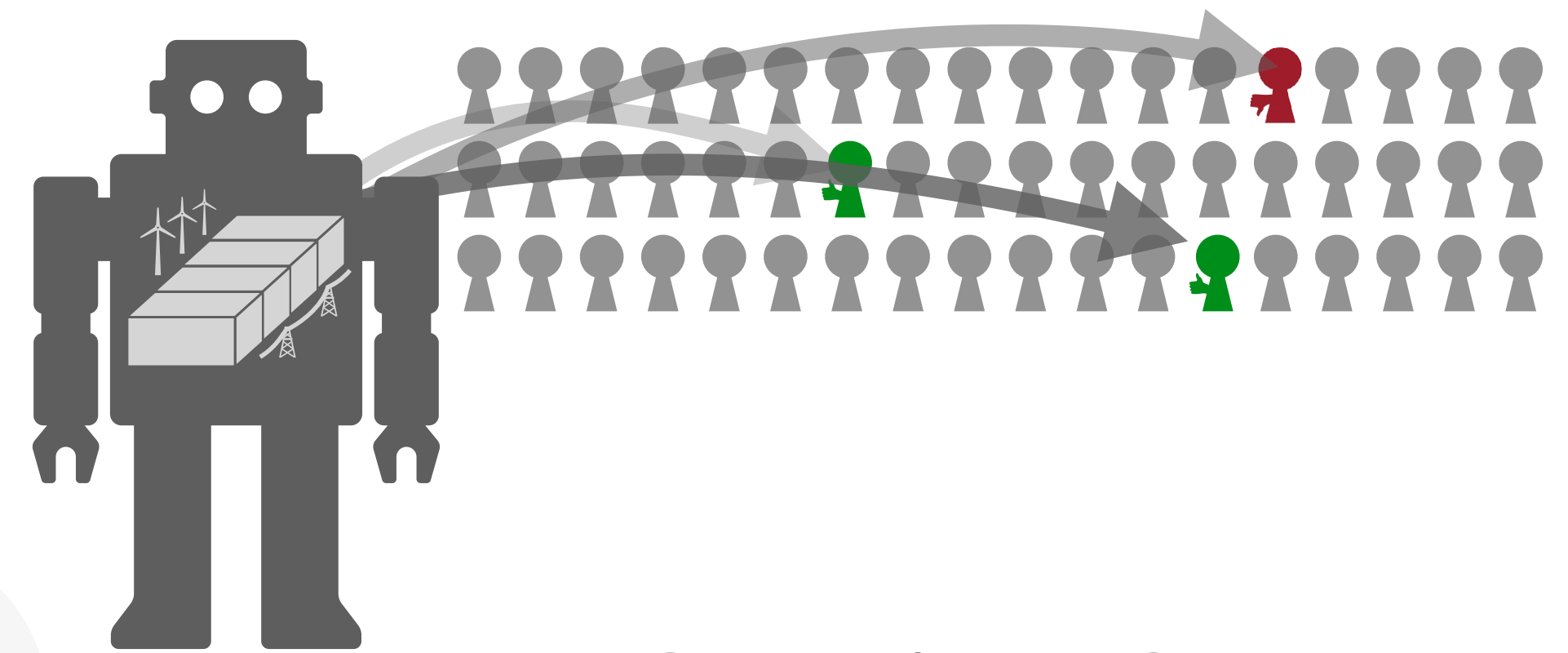
# Classical Hardness

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$$S \geq \Omega(Q_C/Q^2)$$



For ML tasks,

$$Q = \text{poly}(\log N), Q_C = \Theta(N^{0.99})$$

$$\implies S \geq \Omega(N^{0.99})$$

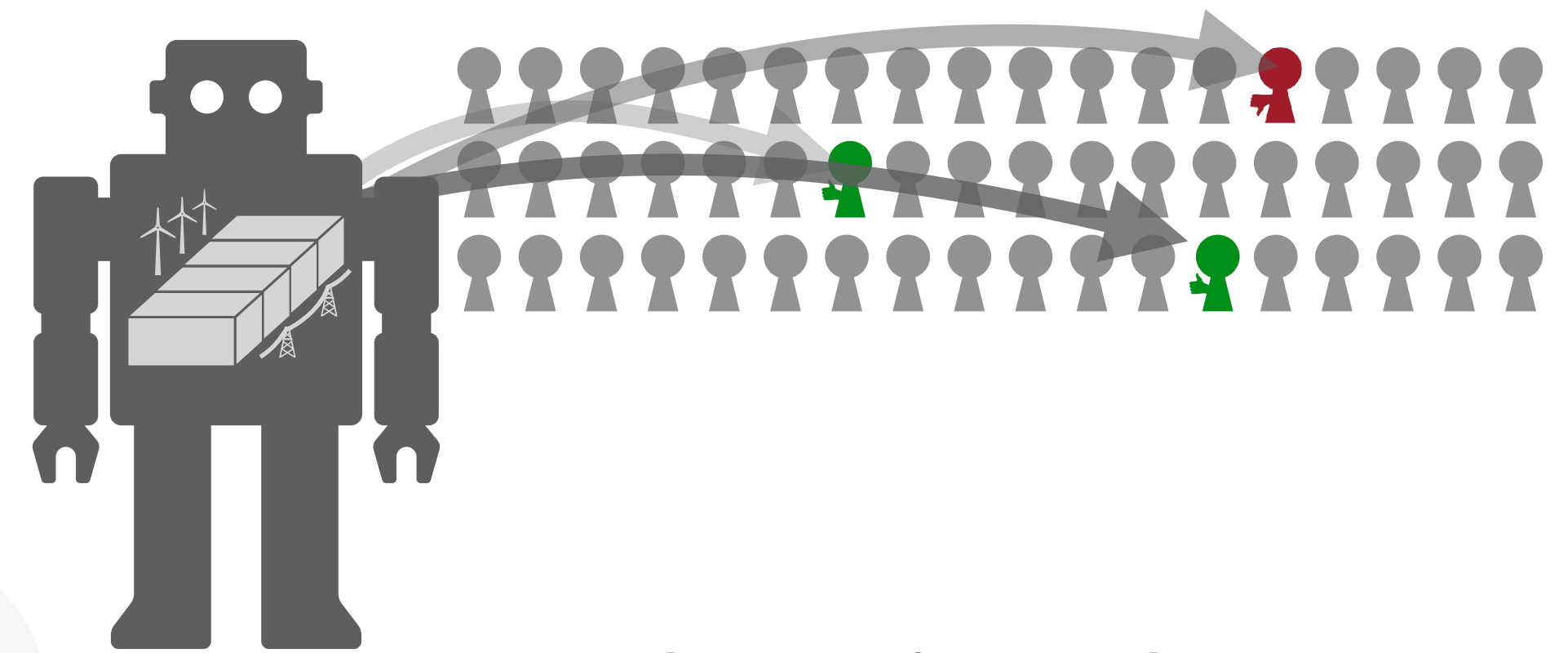
# Classical Hardness

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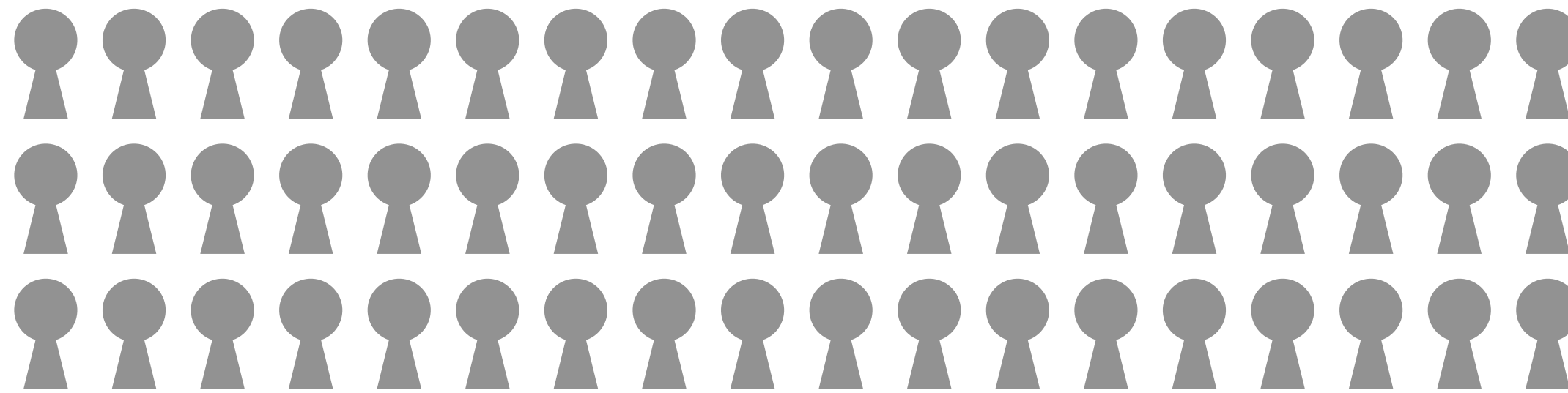
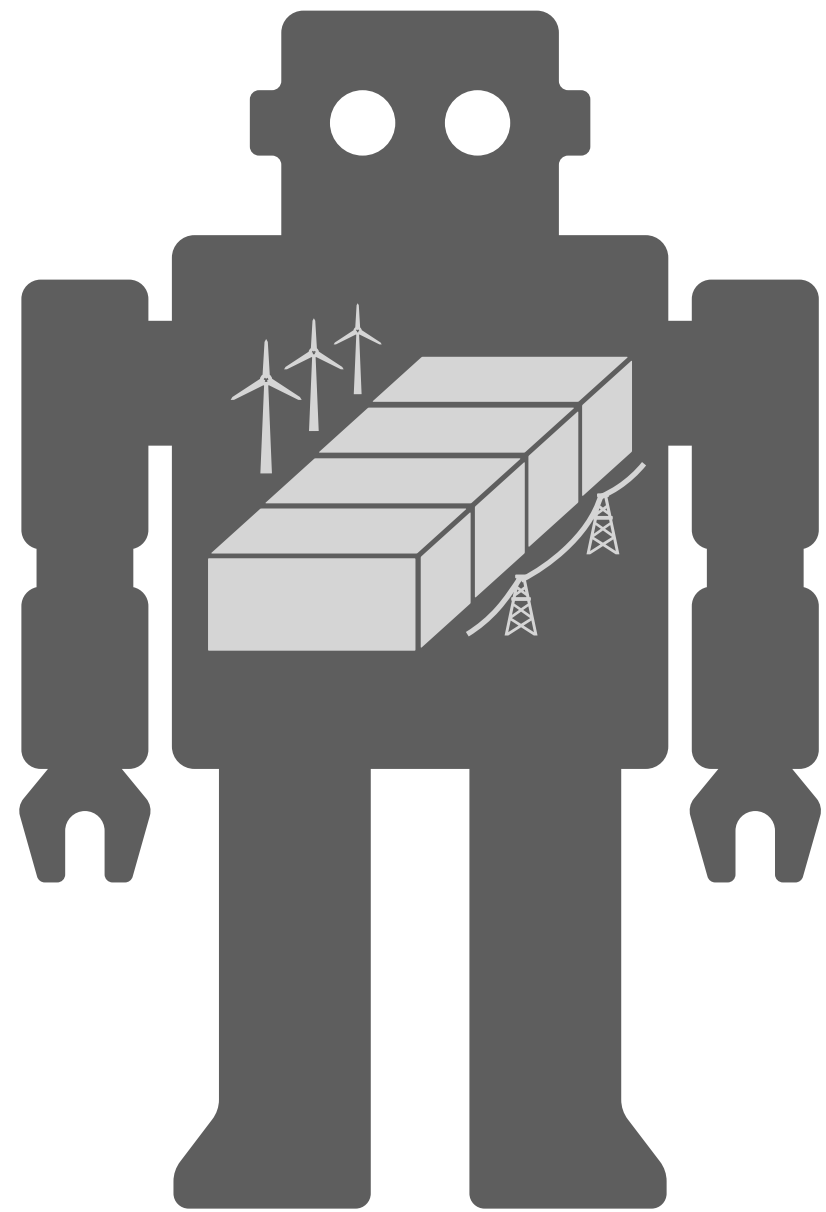
### Classical Query

### Sample-space lower bound

$$MS \geq \Omega(NQ_C)$$

#sample  $M = \Theta(NQ^2)$  from QOS

# Query Problem

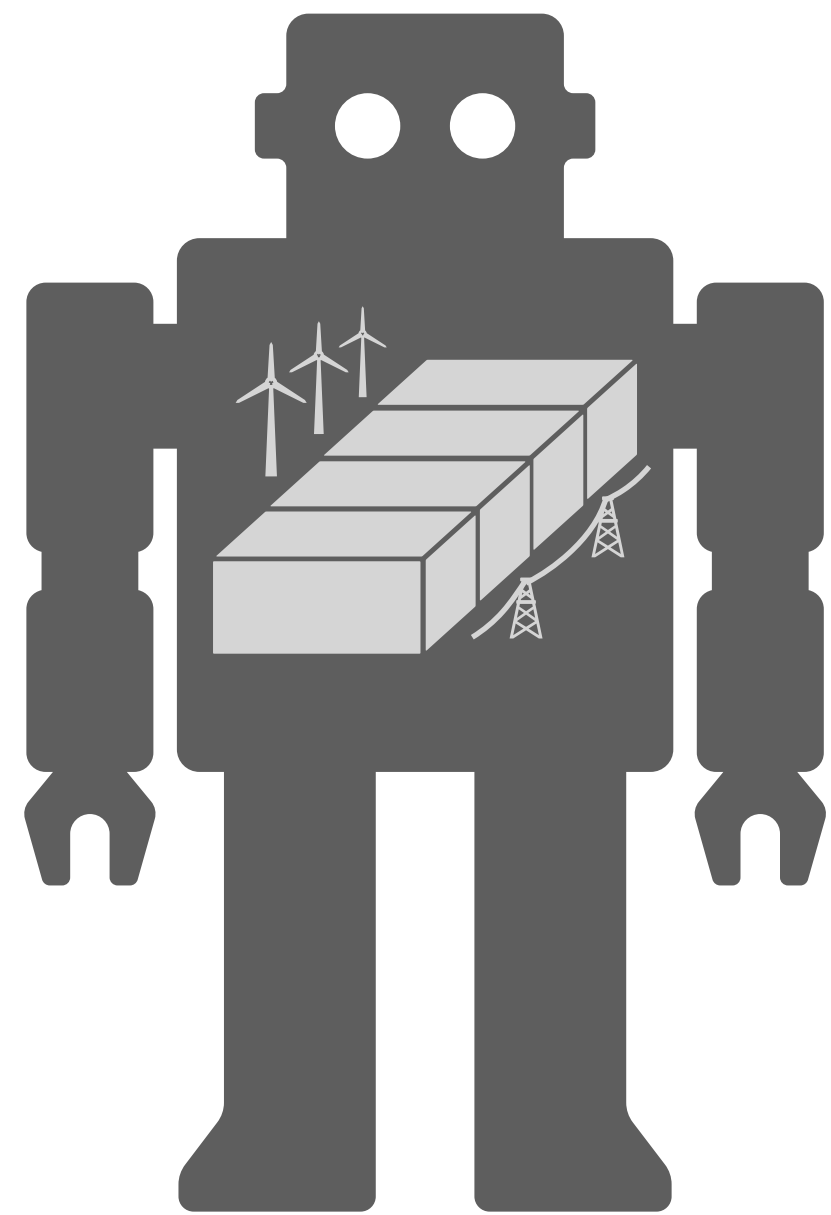


$N$  users



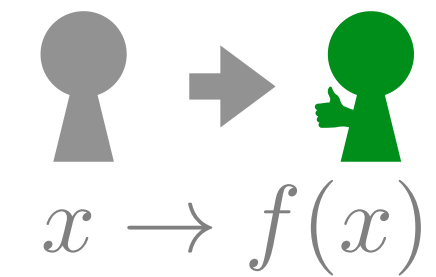
**Goal**  
rate the movie


# Query Problem



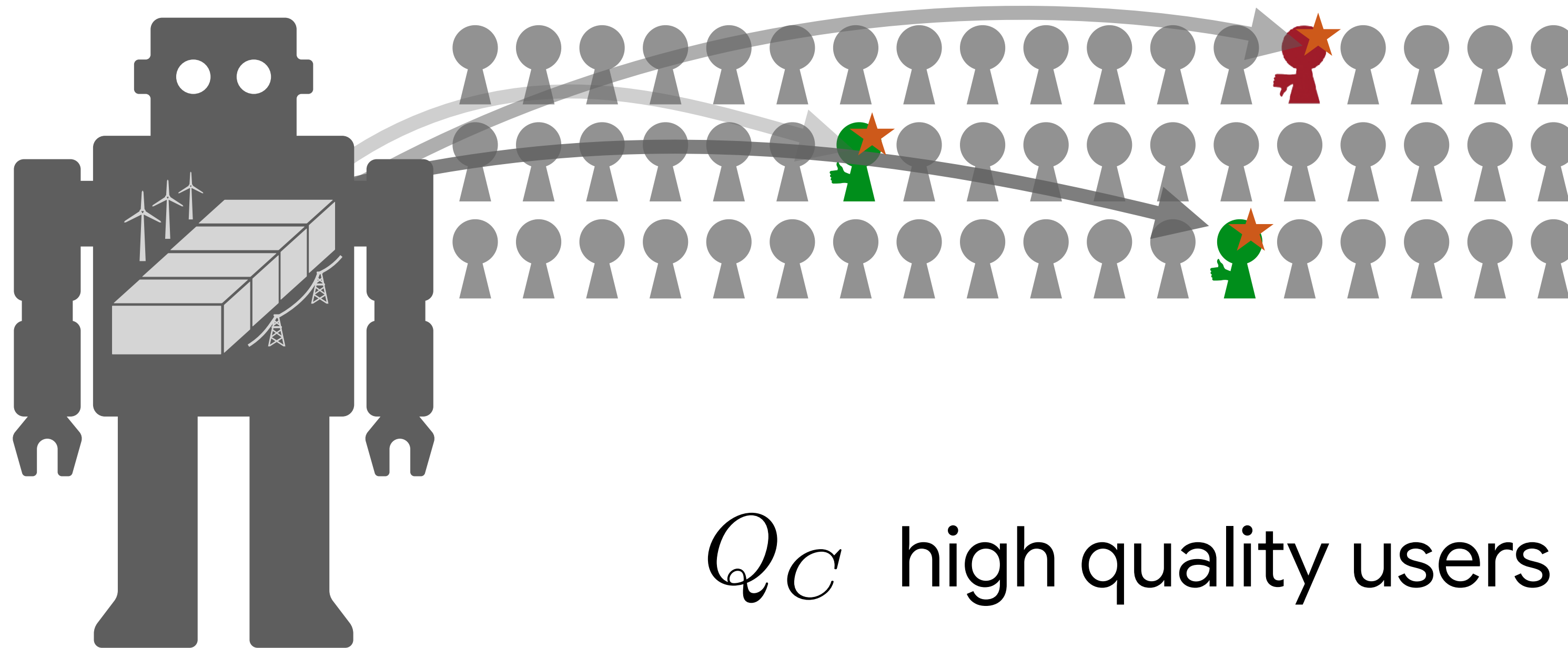
$N$  users

[oracle]



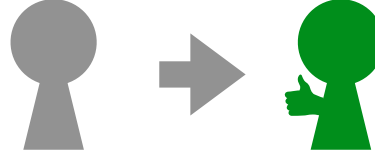
$Q_C$  high quality users 

# Query Problem



$N$  users

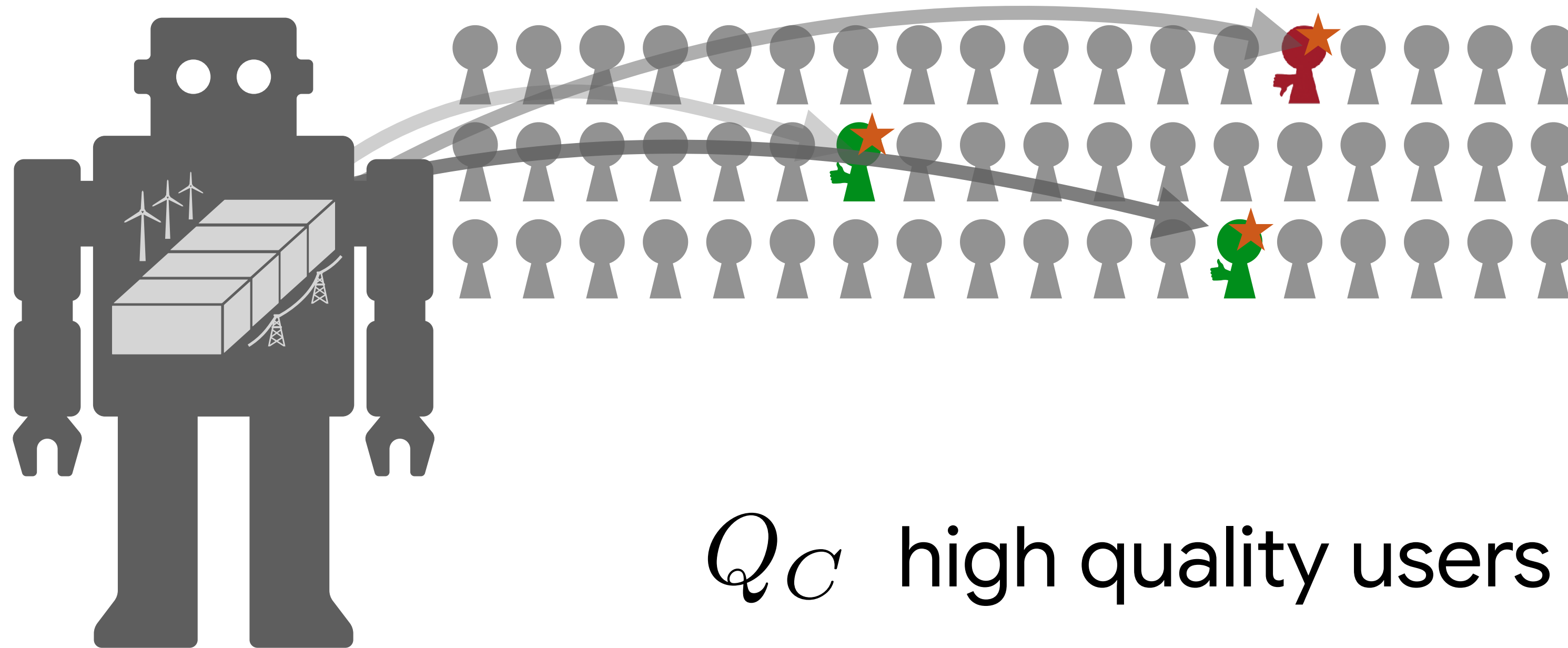
[oracle]

  
 $x \rightarrow f(x)$

$Q_C$  high quality users 

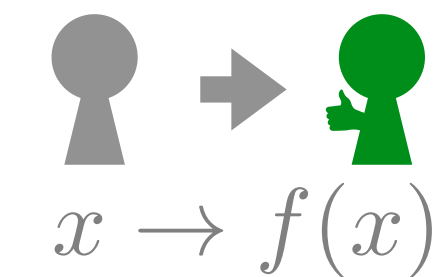
querying them suffices to determine the goal  
[classical query complexity]

# Query Problem



$N$  users

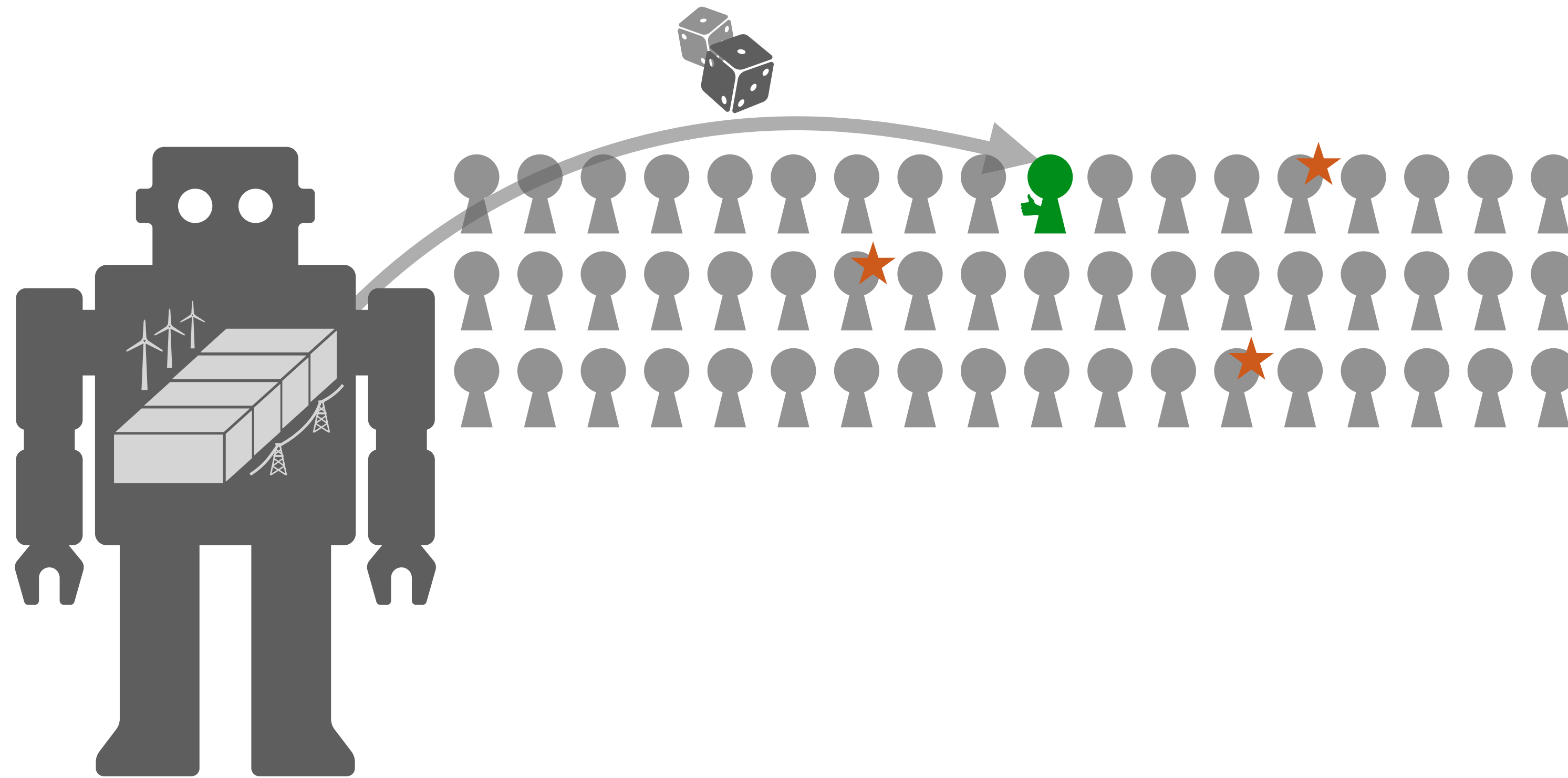
[oracle]



$Q_C$  high quality users 

amount of info needed to estimate some  
property of the oracle [classical query complexity]

# Oracle Property Estimation



random data  
from  $N$  users

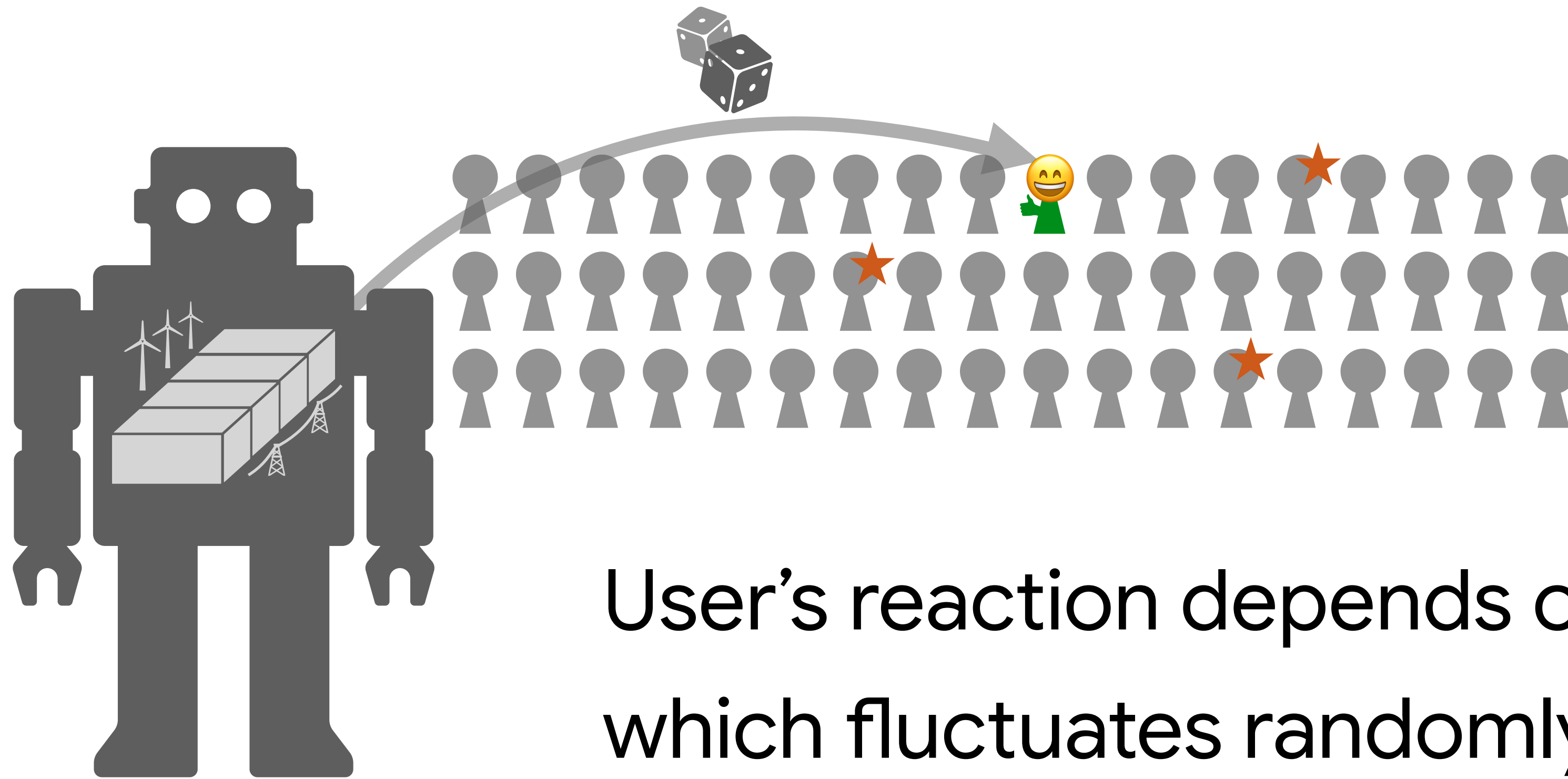
$$z = (x, f(x))$$



$Q_C$

info needed to estimate  
oracle property

# Noisy Oracle Property Estimation (NOPE)



noisy data  
from  $N$  users

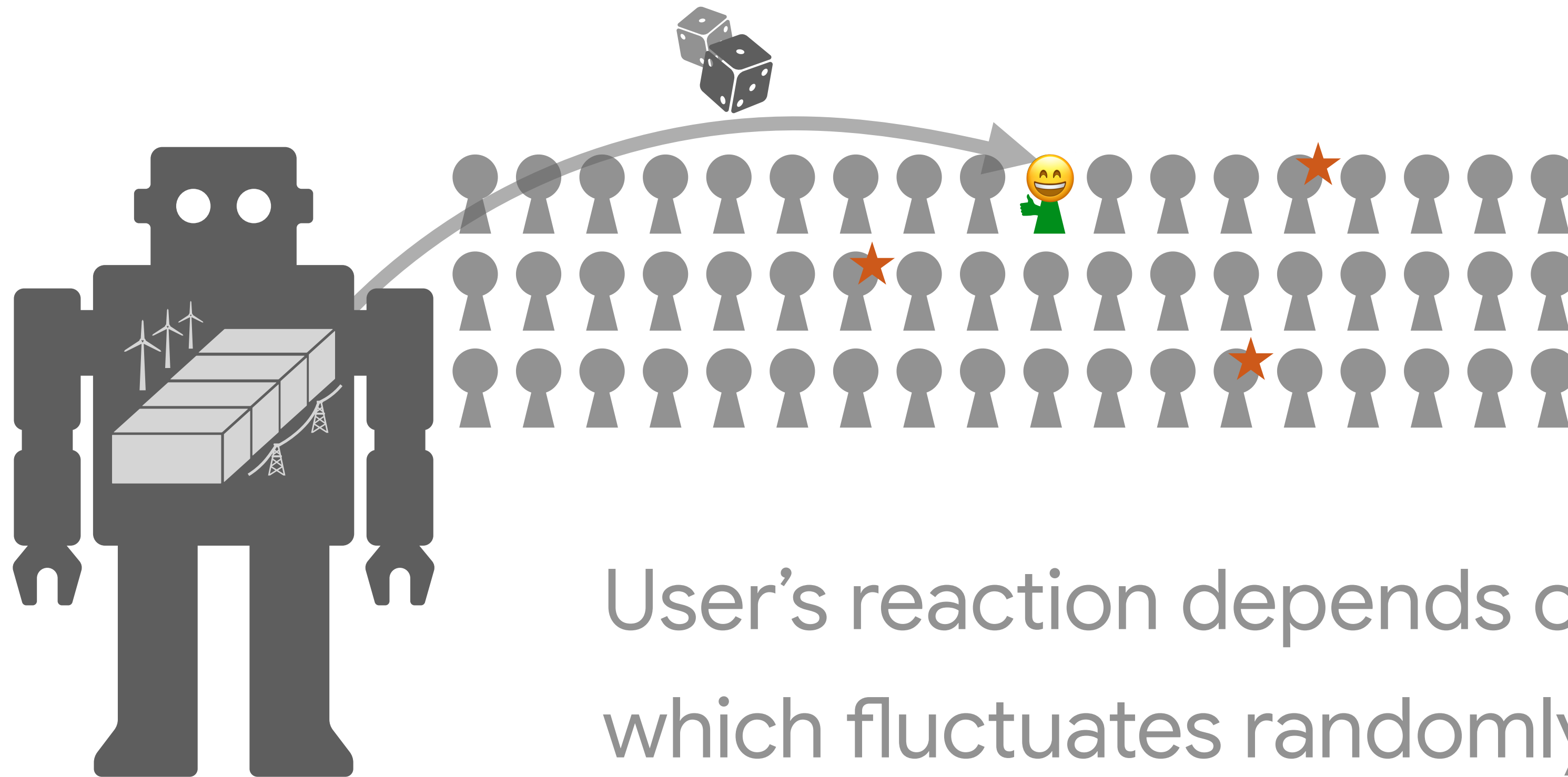
$$z = (x, f_{\alpha}(x), \alpha) \text{ 🤗}$$

User's reaction depends on their mood,  
which fluctuates randomly over time.

$Q_C$

info needed to estimate  
oracle property

# Noisy Oracle Property Estimation (NOPE)



noisy data  
from  $N$  users

$$z = (x, f_\alpha(x), \alpha) \text{ 😊}$$

User's reaction depends on their mood,  
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$Q_C$

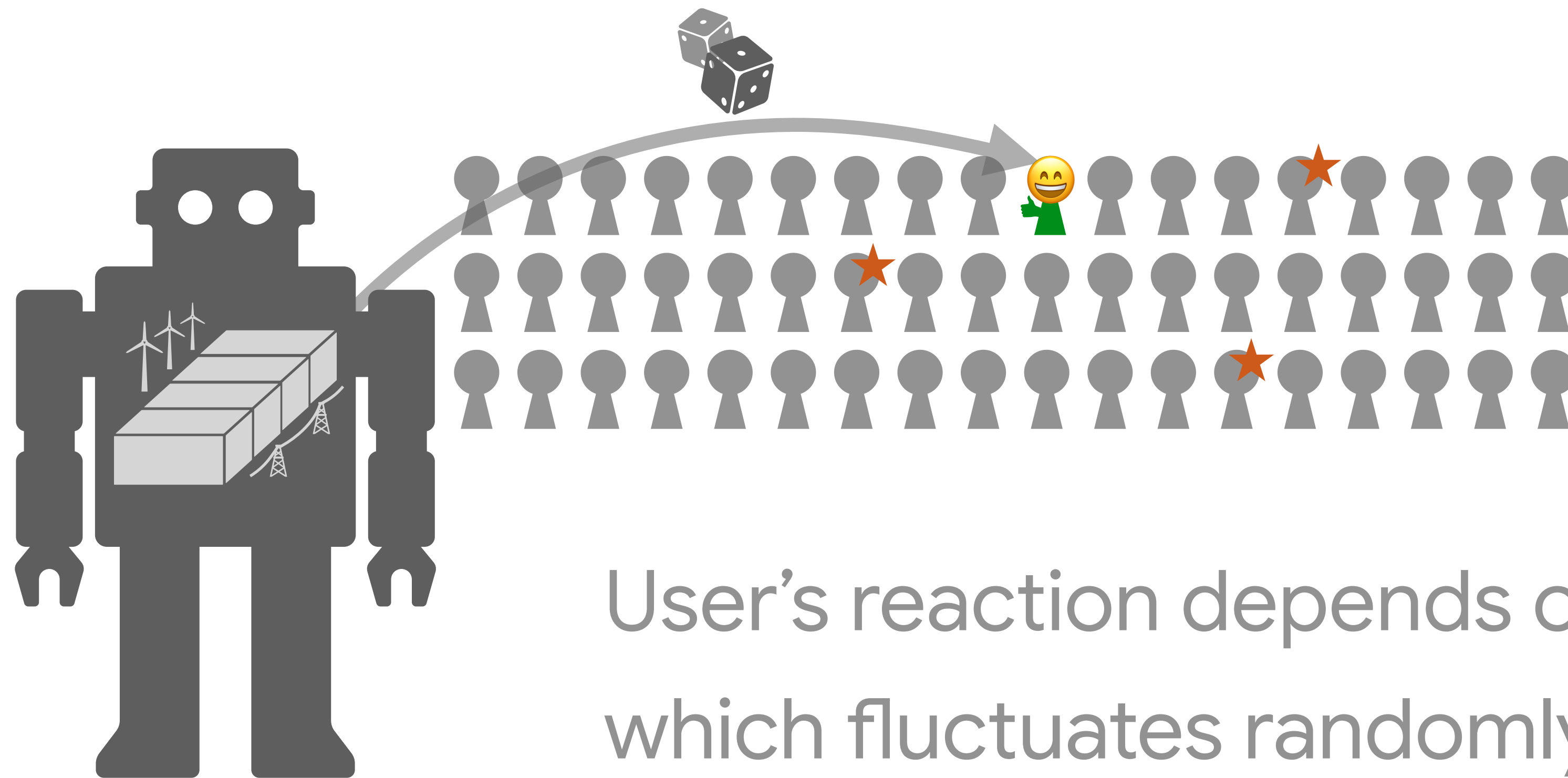
info needed to estimate  
oracle property

Need to gather both for accurate feedback. 😊 😞 → 🟢

$$f_1(x), f_2(x) \rightarrow f(x)$$

via low-discrepancy  
encoding

# Noisy Oracle Property Estimation (NOPE)



noisy data  
from  $N$  users

$$z = (x, f_\alpha(x), \alpha) \text{ 😊}$$

User's reaction depends on their mood,  
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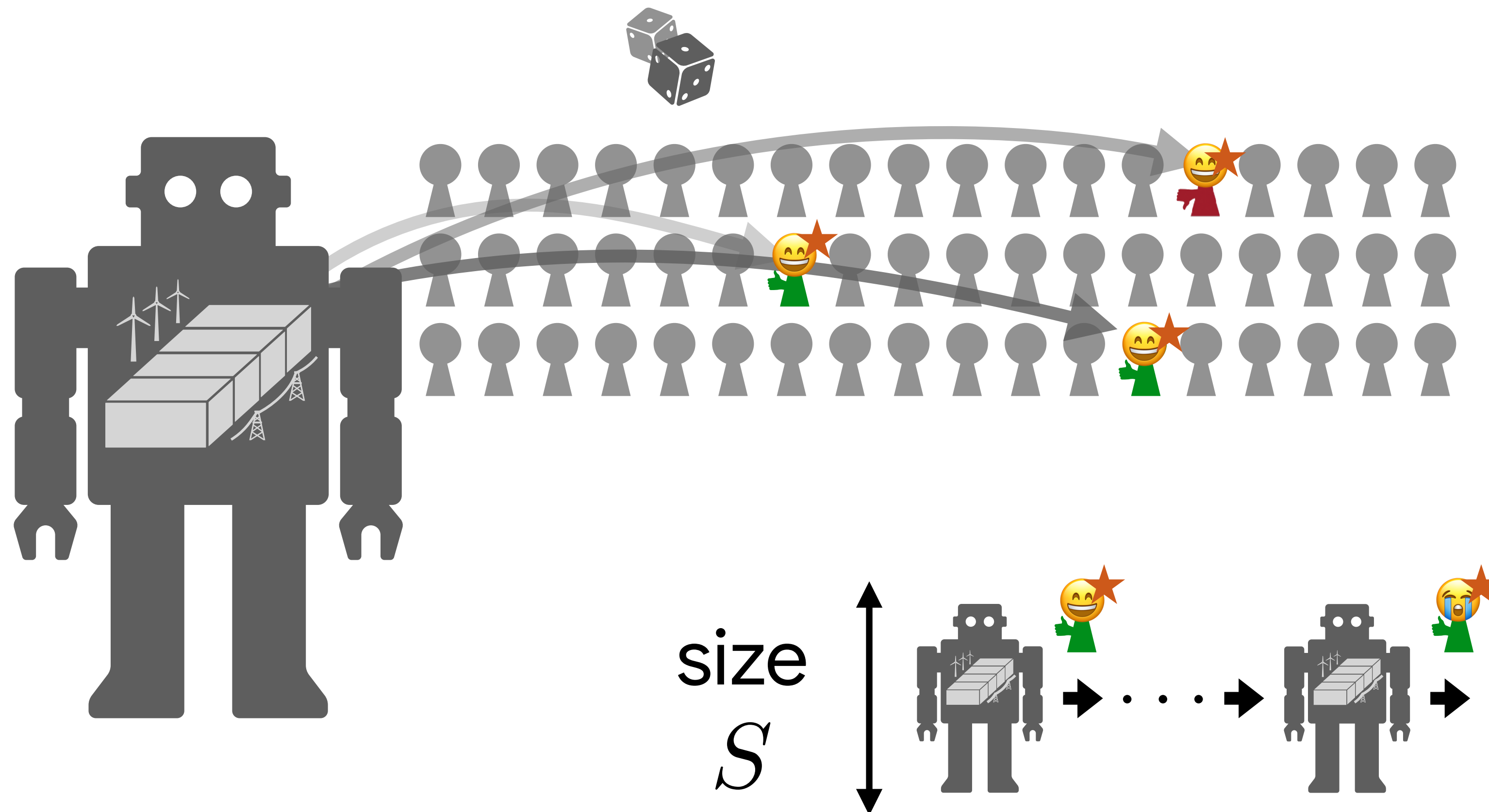
$Q_C$

info needed to estimate  
oracle property

Need to gather both for accurate feedback. 😊 😞 → 🟢  
 $f_1(x), f_2(x) \rightarrow f(x)$

That takes  $\Theta(N)$  time/samples.  $p(x) = 1/N$

# Noisy Oracle Property Estimation (NOPE)



$Q_C$

info needed to estimate  
oracle property

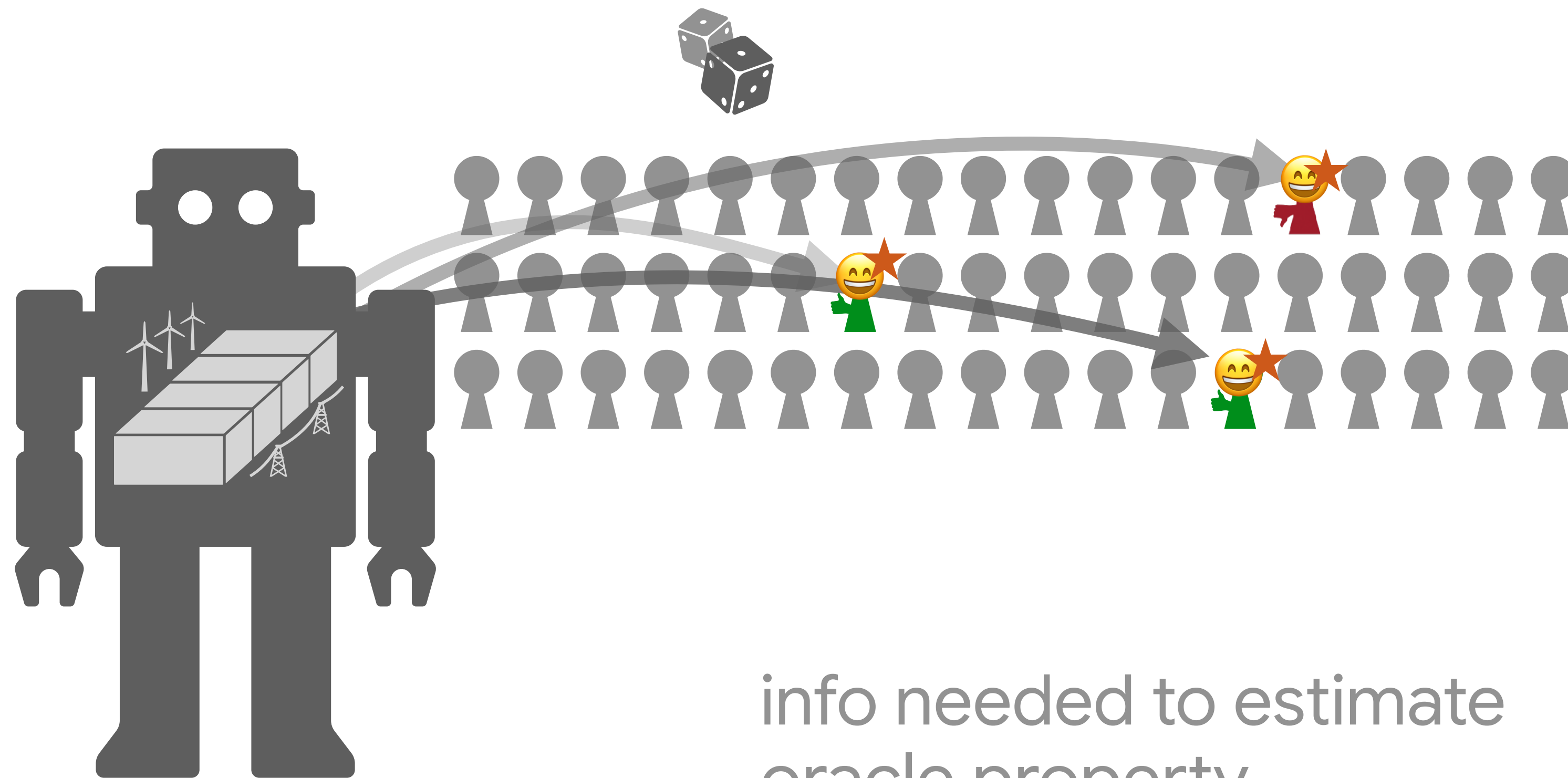
noisy data  
from  $N$  users

$$z = (x, f_\alpha(x), \alpha) \text{ 🧑}$$

$$\text{info} \leq O\left(\frac{M}{N}\right) \cdot S$$

# rounds

# Noisy Oracle Property Estimation (NOPE)



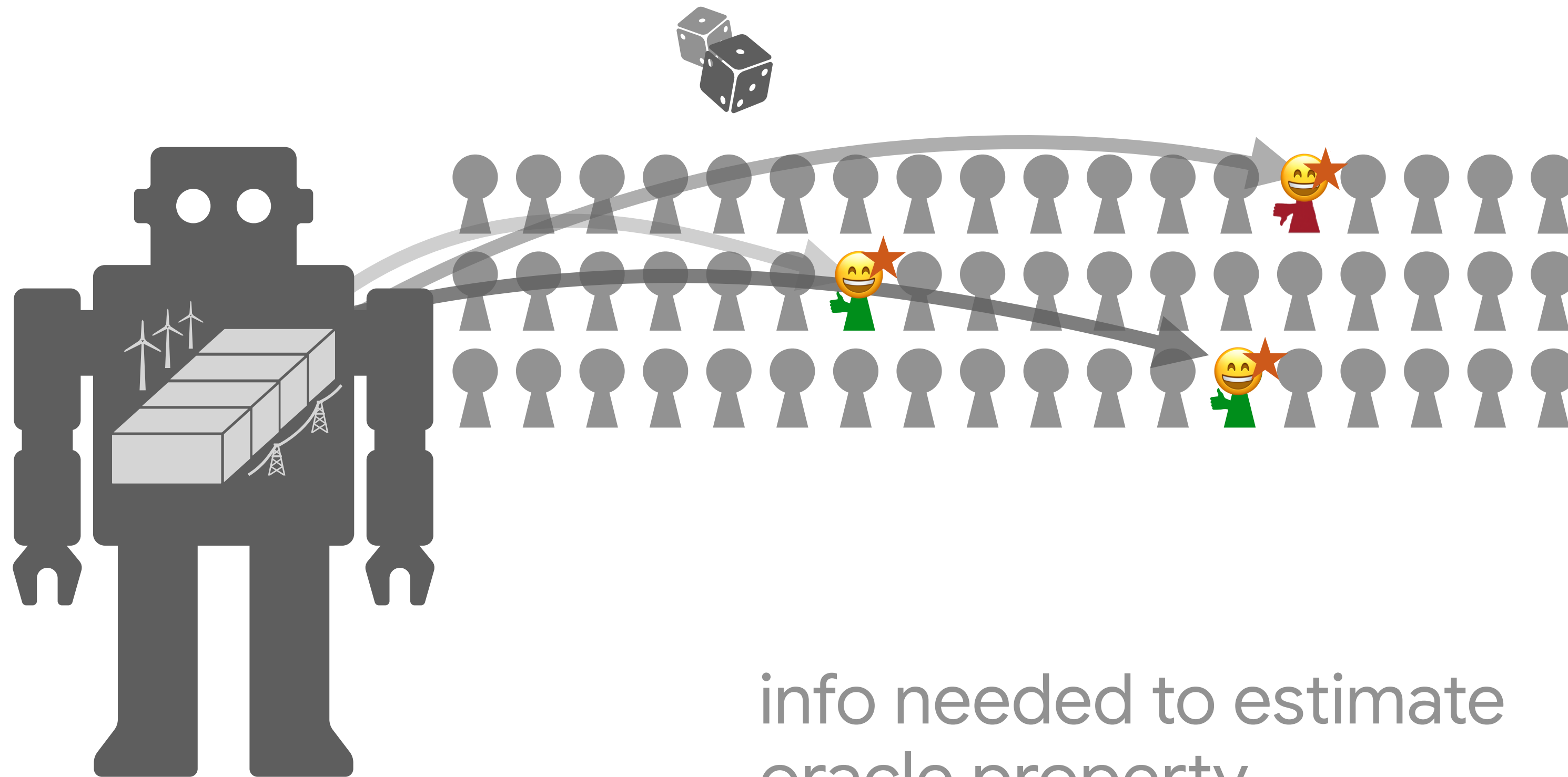
info needed to estimate  
oracle property

noisy data  
from  $N$  users

$$z = (x, f_\alpha(x), \alpha) \text{ 🌟}$$

$$Q_C \leq \text{info} \leq O\left(\frac{M}{N}\right) \cdot S$$

# Noisy Oracle Property Estimation (NOPE)



noisy data  
from  $N$  users

$$z = (x, f_\alpha(x), \alpha) \text{ 🌟}$$

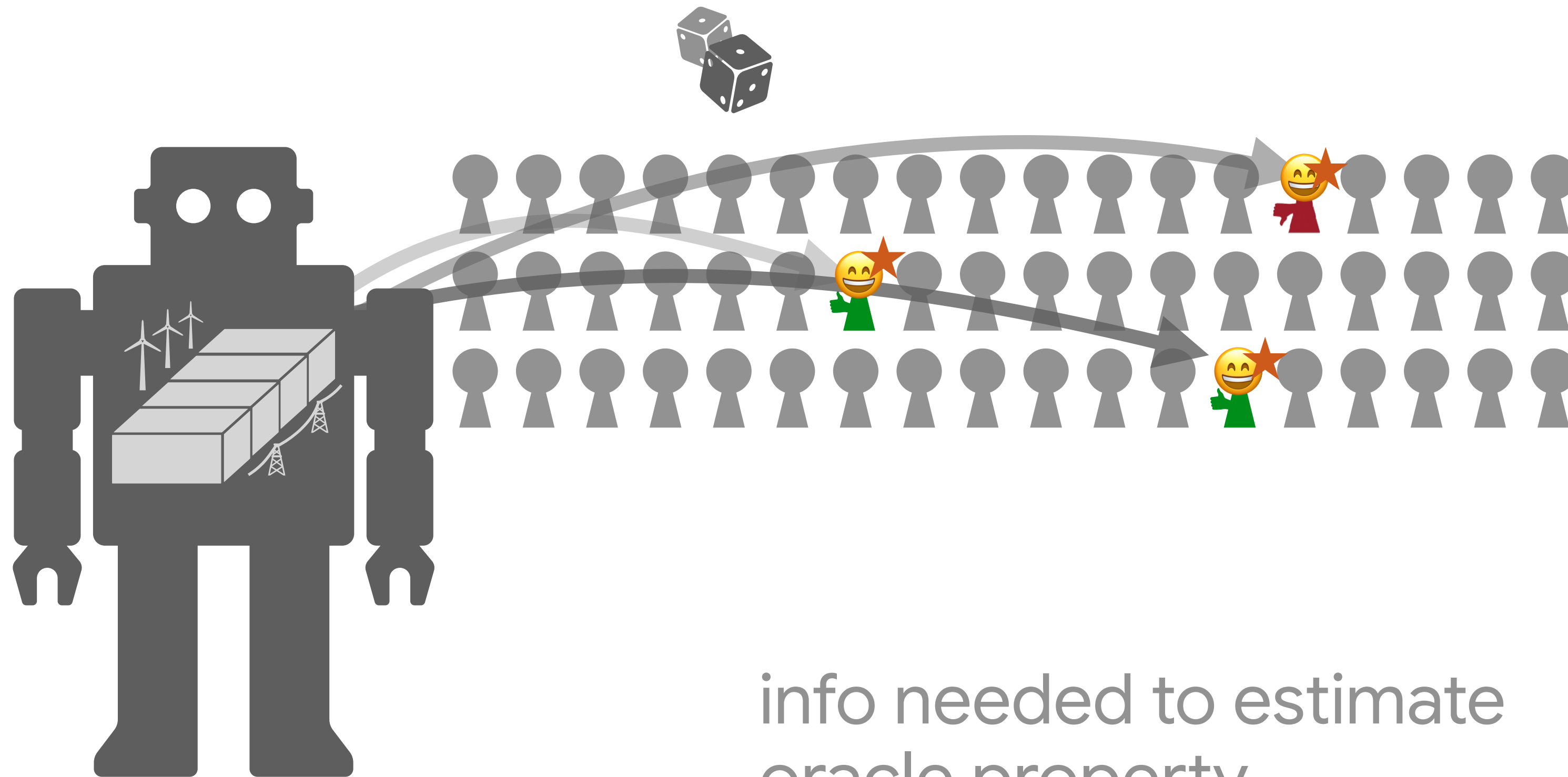
info needed to estimate  
oracle property

$$Q_C \leq \text{info} \leq O\left(\frac{M}{N}\right) \cdot S$$

Sample-space lower bound

$$MS \geq \Omega(NQ_C)$$

# Noisy Oracle Property Estimation (NOPE)



noisy data  
from  $N$  users

$$z = (x, f_\alpha(x), \alpha) \text{ 🌟}$$

info needed to estimate  
oracle property

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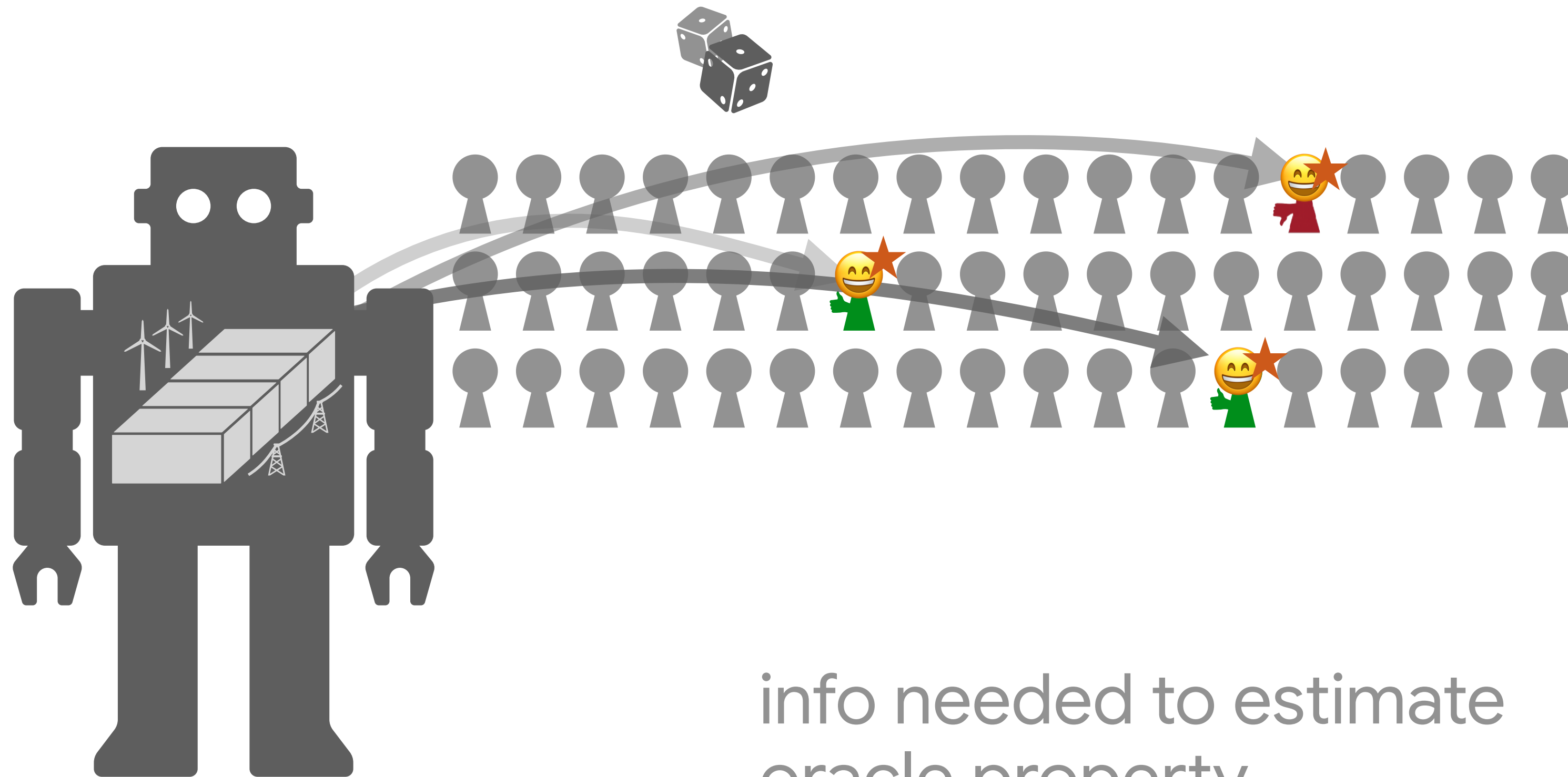
Theorem (Classical Hardness)

$$S \geq \Omega(Q_C/Q^2)$$

$$MS \geq \Omega(NQ_C)$$

#sample  $M = \Theta(NQ^2)$  from QOS

# Noisy Oracle Property Estimation (NOPE)



noisy data  
from  $N$  users

$$z = (x, f_\alpha(x), \alpha) \text{ 🌟}$$

info needed to estimate  
oracle property

$$Q_C \leq \text{info} \leq O\left(\frac{M}{N}\right) \cdot S$$

Theorem (Classical Hardness)

$$S \geq \Omega(N^{1-\zeta})$$

NOPE of Correlation

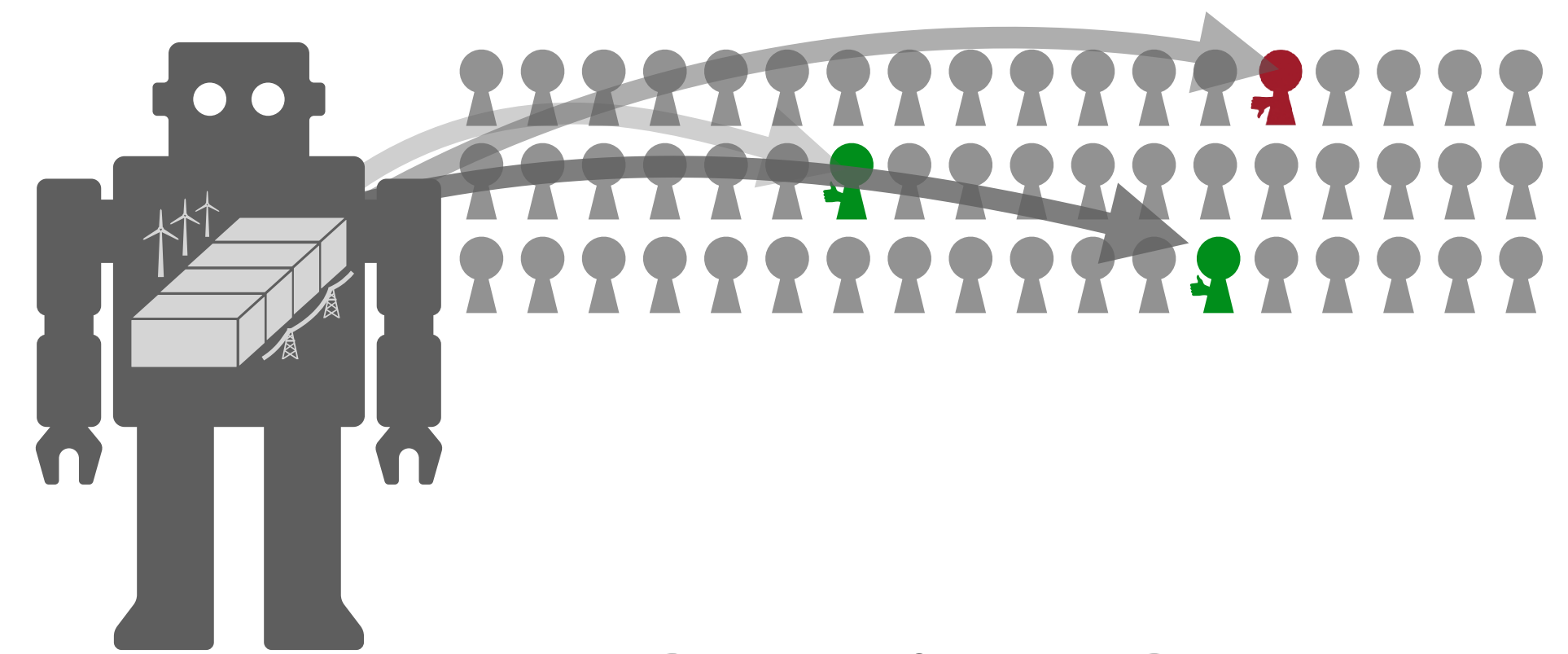
$$Q_C = \Omega(N^{1-\zeta}), Q = O(1)$$

# Classical Hardness

1. Connect query separation to memory advantage

$$S \geq \Omega(N^{0.99})$$

2. Insufficient memory must be compensated by more samples



Classical Query

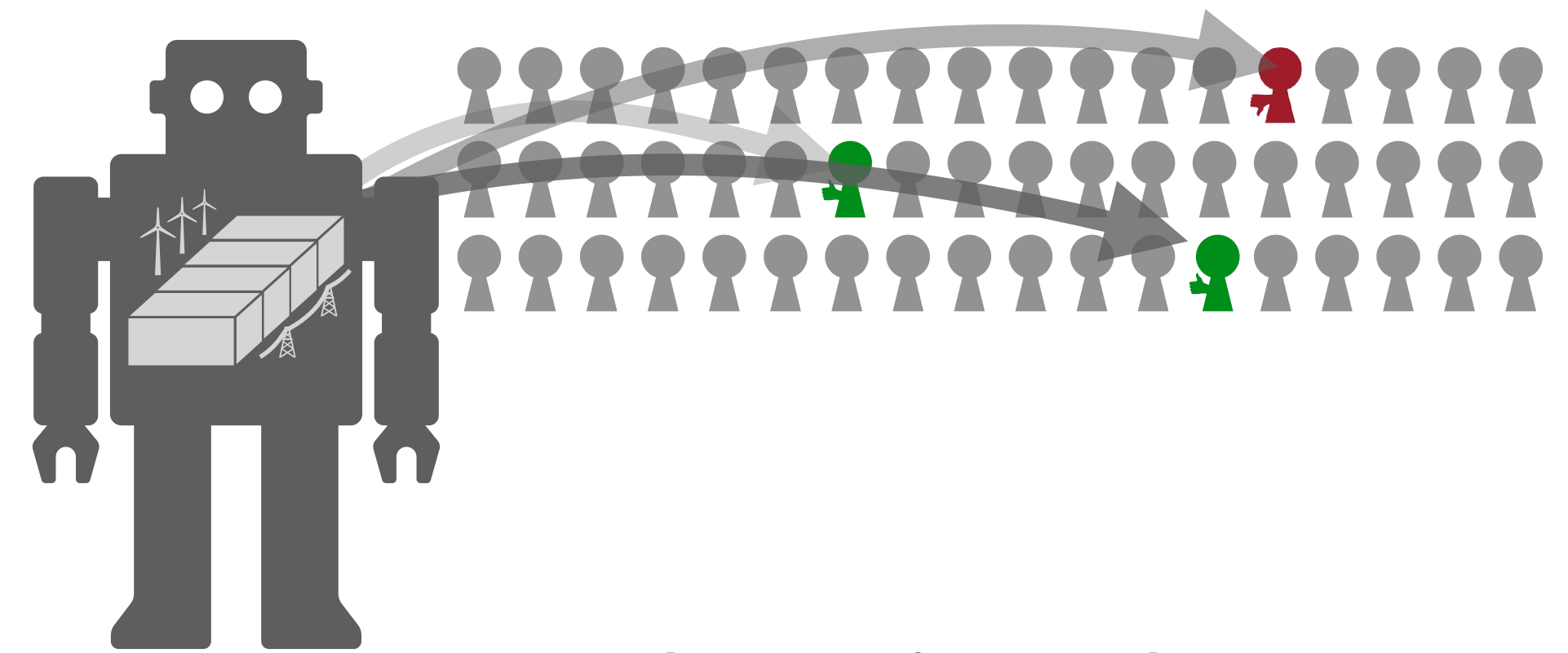
# Classical Hardness

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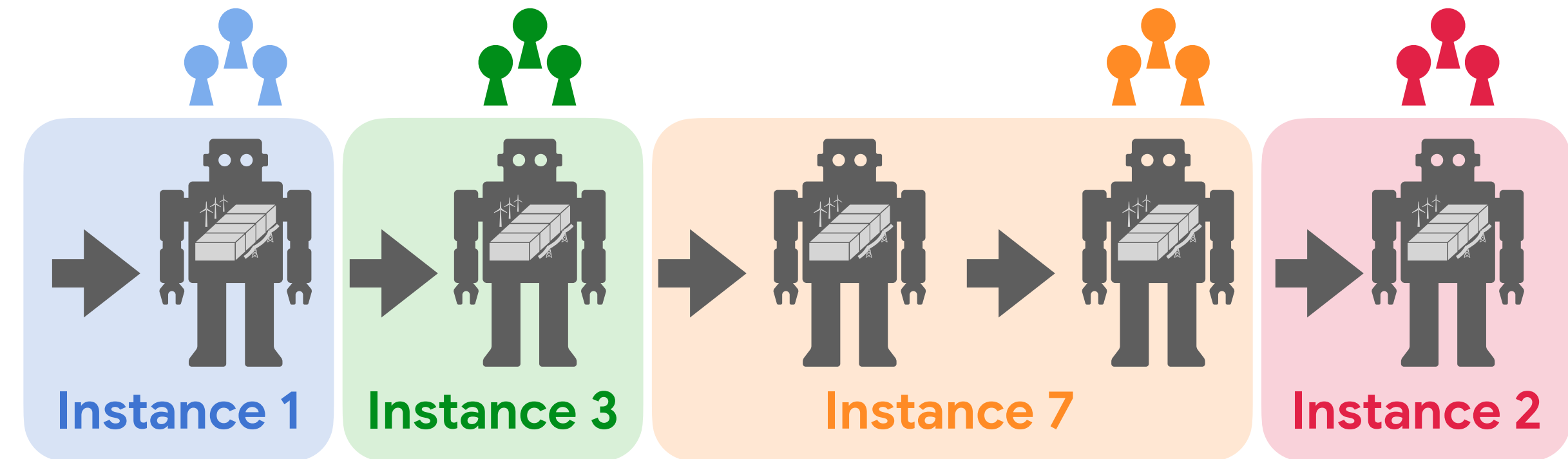
NOPE: if  $S \leq O(N^{0.99})$ , then progress  $\leq 0.1$



# Classical Hardness

1. Connect query separation to memory advantage

$$S \geq \Omega(N^{0.99})$$



2. Insufficient memory must be compensated by more samples

simultaneously solve indep. instances of NOPE (the XOR)

NOPE: if  $S \leq O(N^{0.99})$ , then progress  $\leq 0.1$

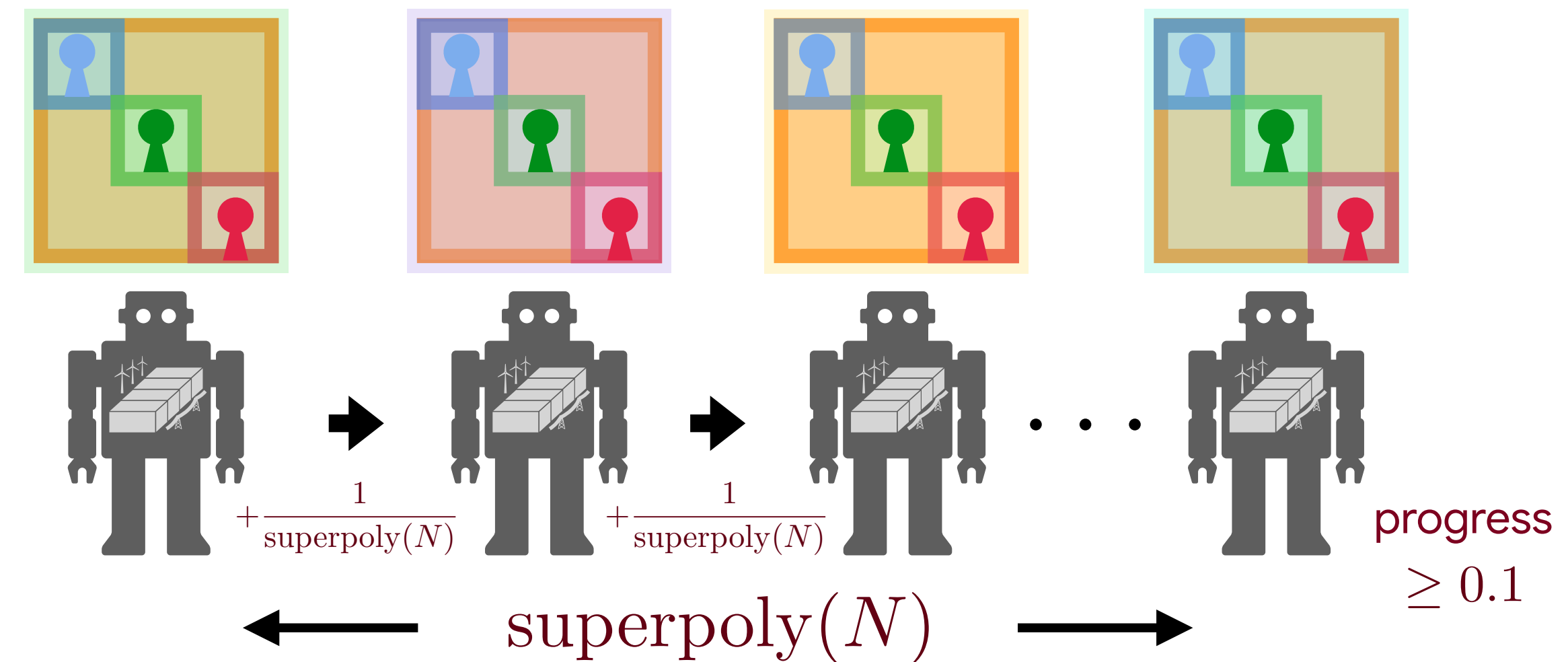
XOR NOPE: if  $S \leq O(N^{0.99})$ , then progress  $\leq (0.1)^{\text{poly}(\log N)} = 1/\text{superpoly}(N)$

# Classical Hardness

1. Connect query separation to memory advantage

$$S \geq \Omega(N^{0.99})$$

2. Insufficient memory must be compensated by more samples



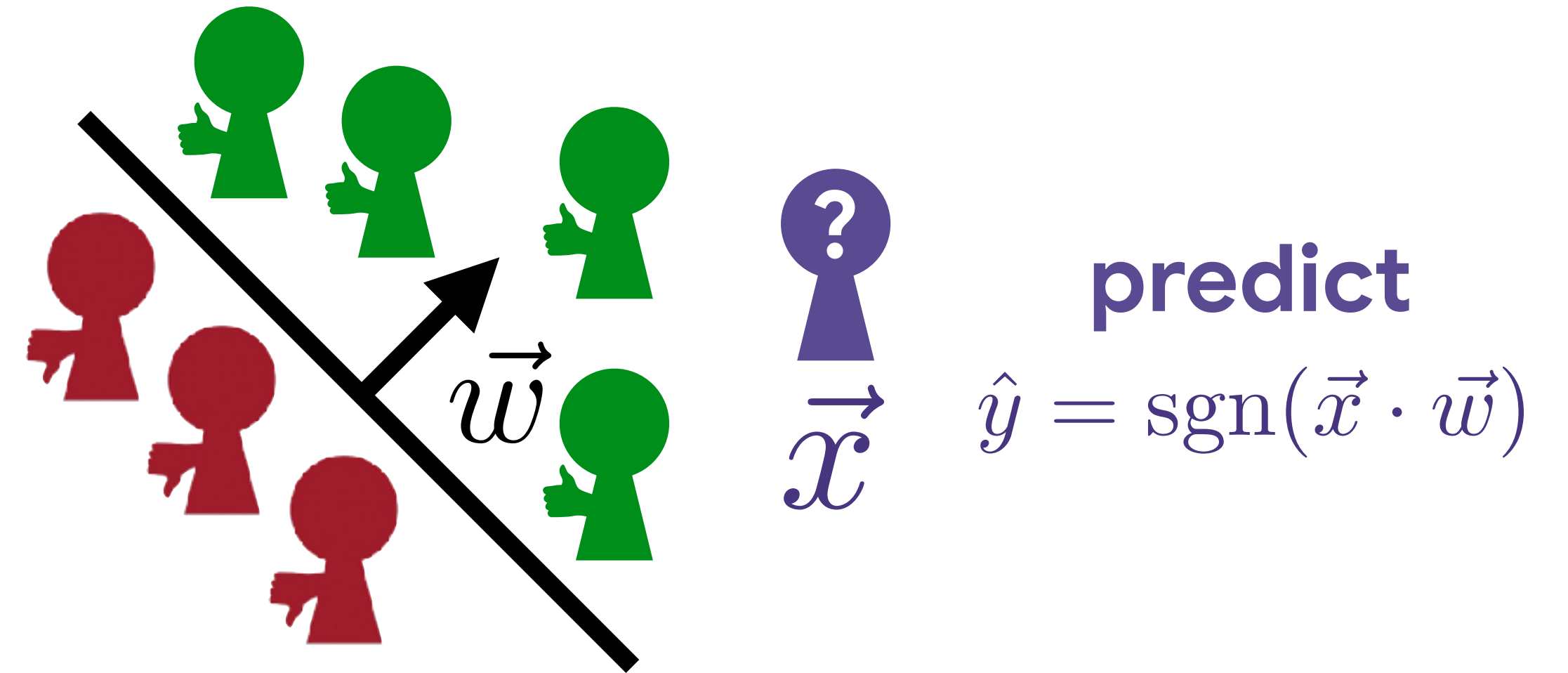
NOPE: if  $S \leq O(N^{0.99})$ , then  $\text{progress} \leq 0.1$

XOR NOPE: if  $S \leq O(N^{0.99})$ , then  $\text{progress} \leq (0.1)^{\text{poly}(\log N)} = 1/\text{superpoly}(N)$

dynamic NOPE: if  $S \leq O(N^{0.99})$ , then requires  $\text{superpoly}(N)$  samples

# Classical Hardness

1. Connect query separation to memory advantage
2. Insufficient memory must be compensated by more samples
3. Embed into ML tasks



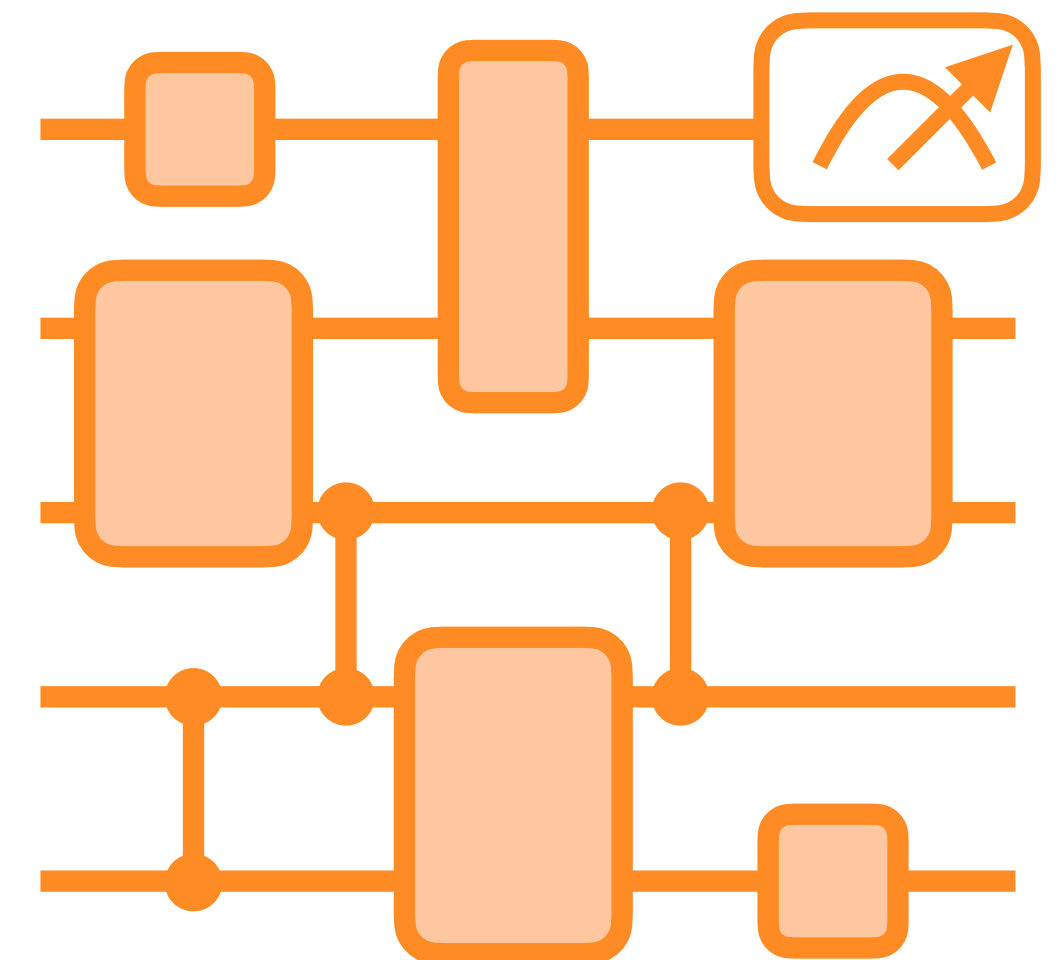
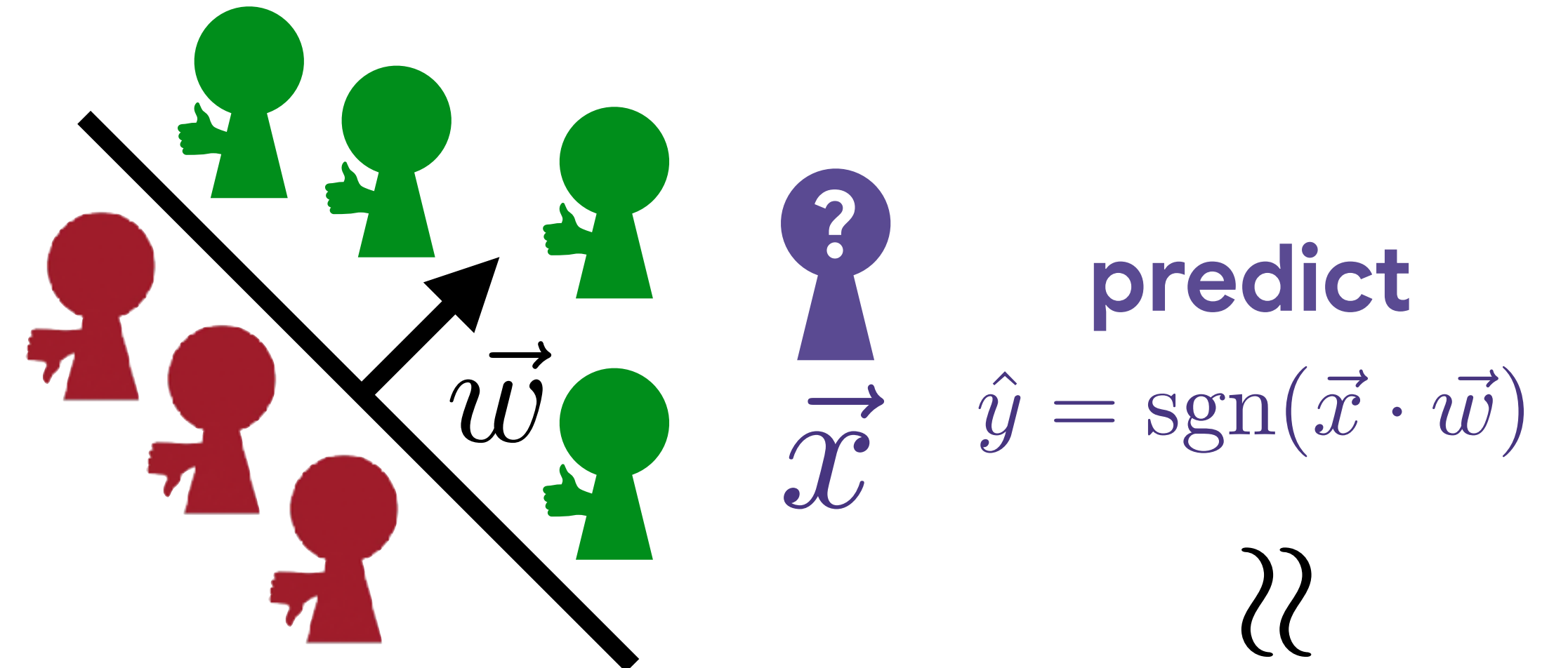
# Classical Hardness

1. Connect query separation to memory advantage

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3. Embed into ML tasks

a. prove BQP hardness of ML tasks



# Classical Hardness

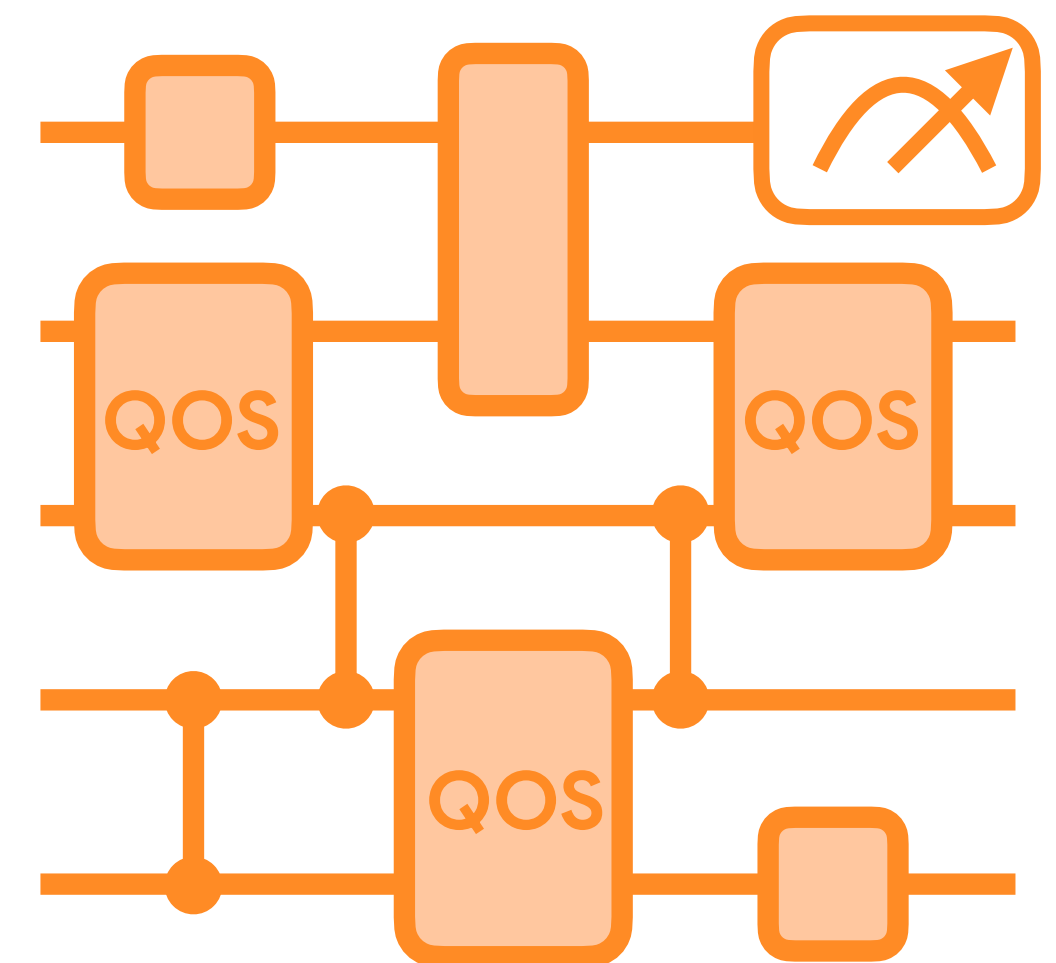
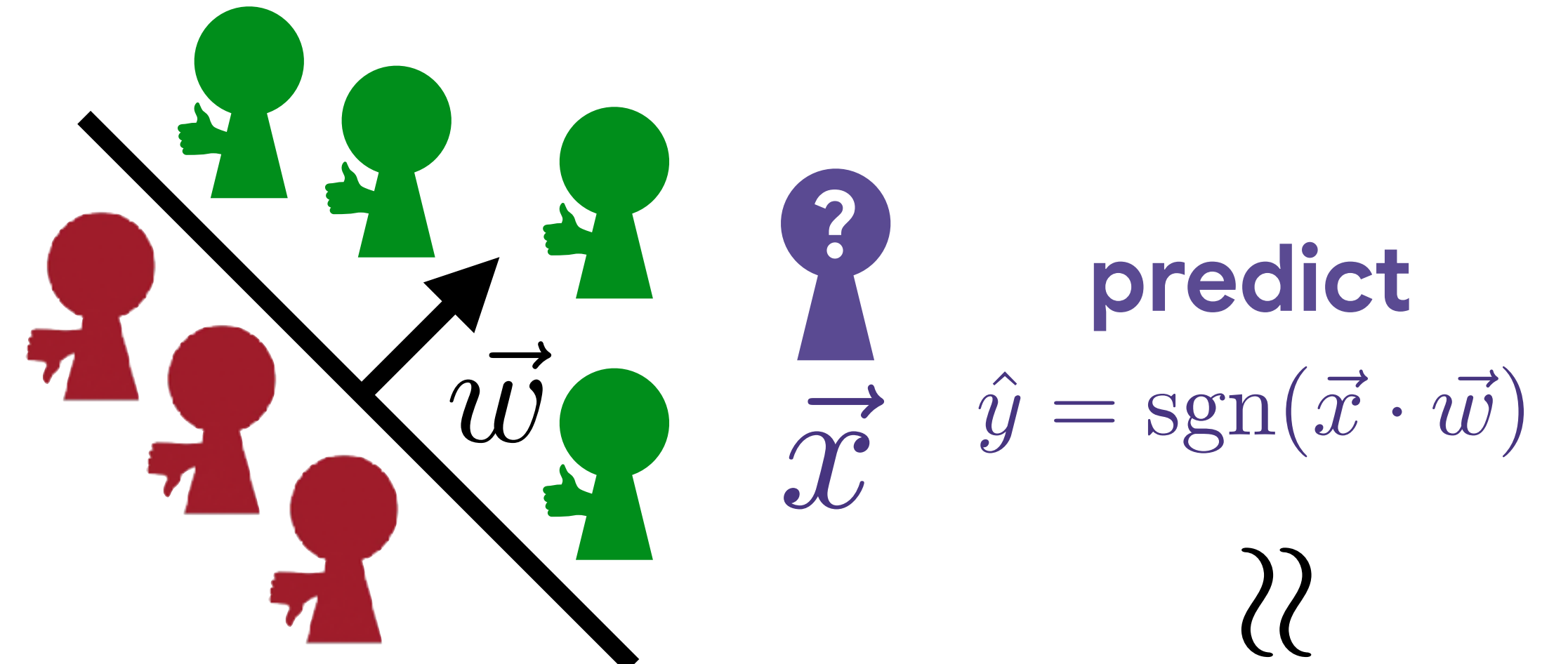
1. Connect query separation to memory advantage

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3. Embed into ML tasks

a. prove BQP hardness of ML tasks

b. embed QOS circuit for dynamic NOPE



# Classical Hardness

1. Connect query separation to memory advantage

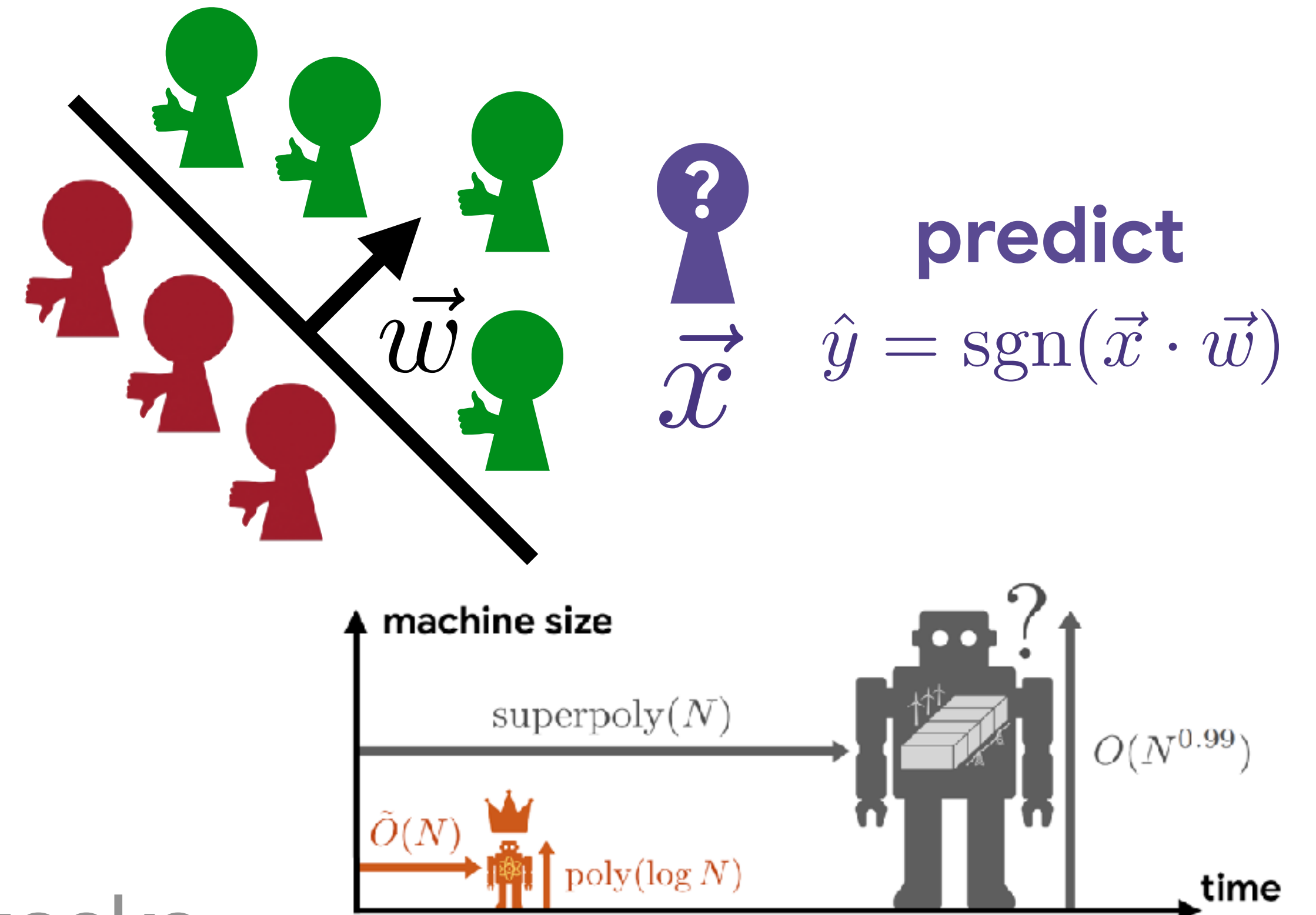
2. Insufficient memory must be compensated by more samples

3. Embed into ML tasks

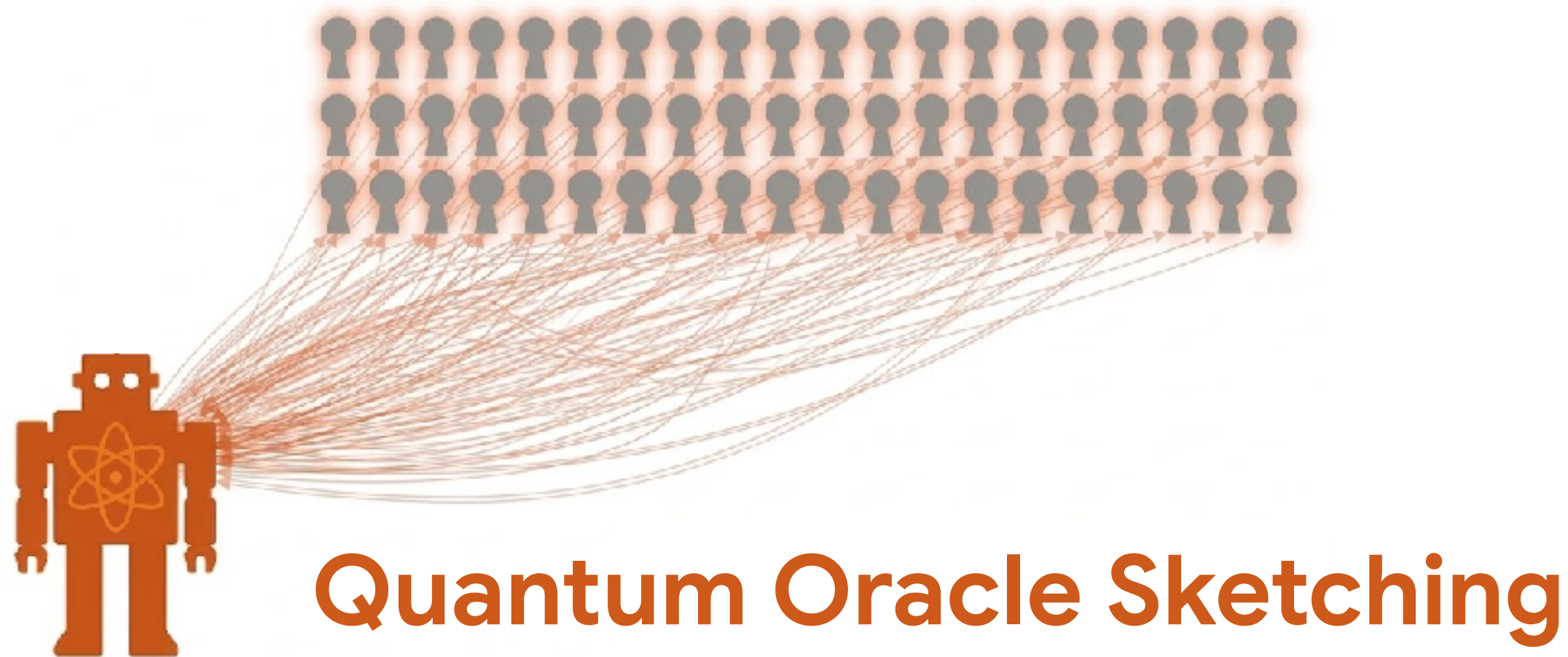
a. prove BQP hardness of ML tasks

b. embed QOS circuit for dynamic NOPE

→ dynamic ML is classically hard

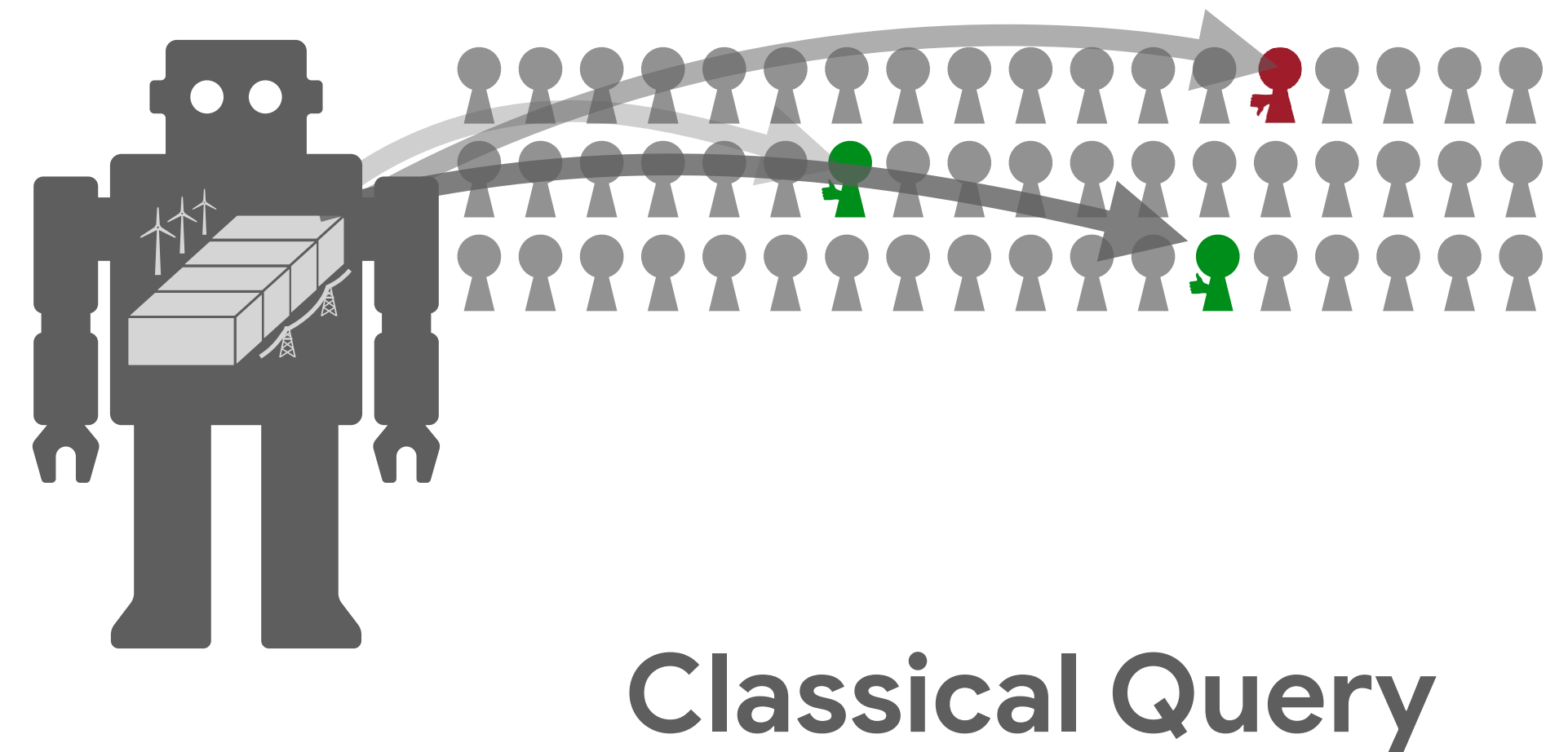


## Quantum Algorithm



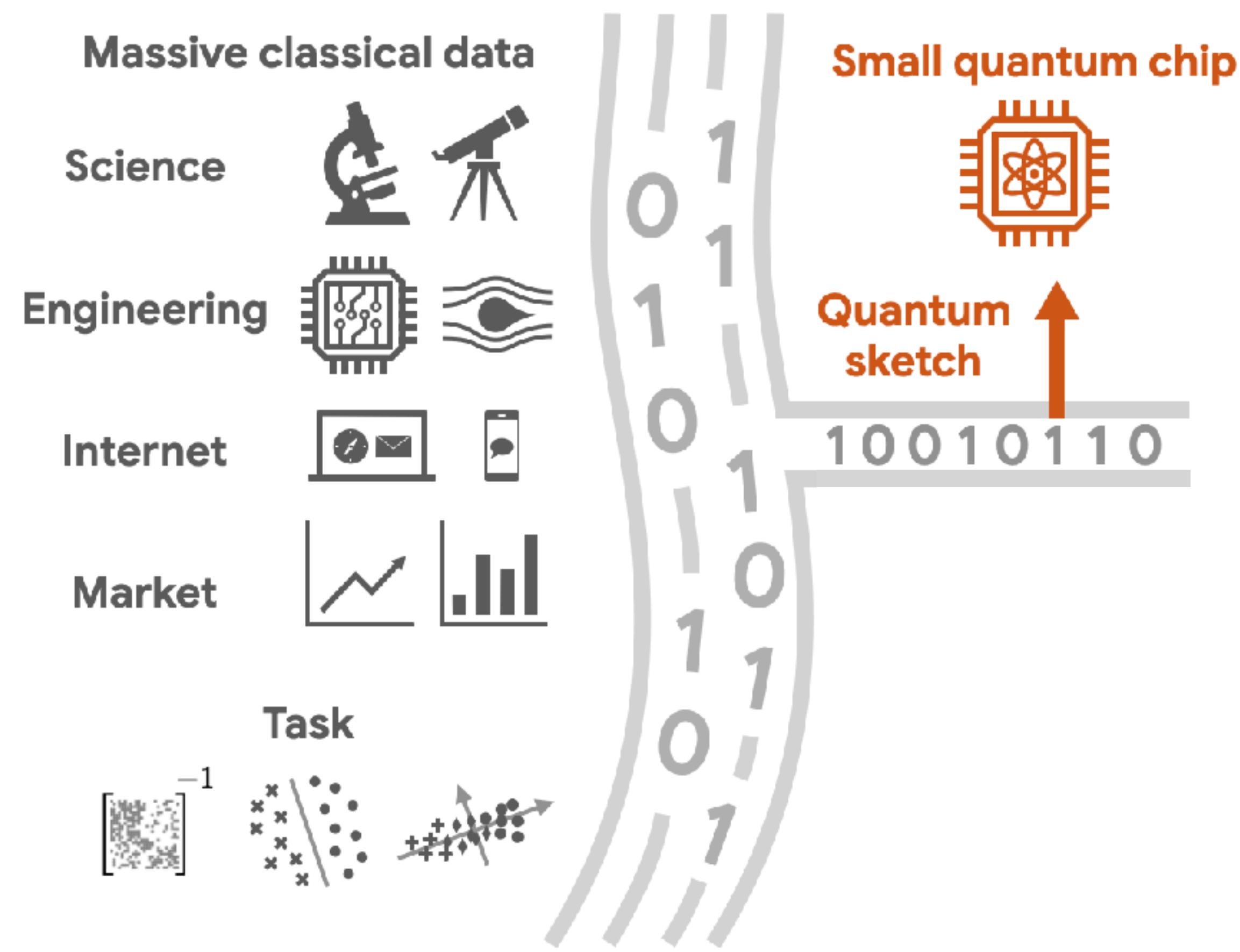
query the classical world in  
quantum superposition

## Classical Hardness



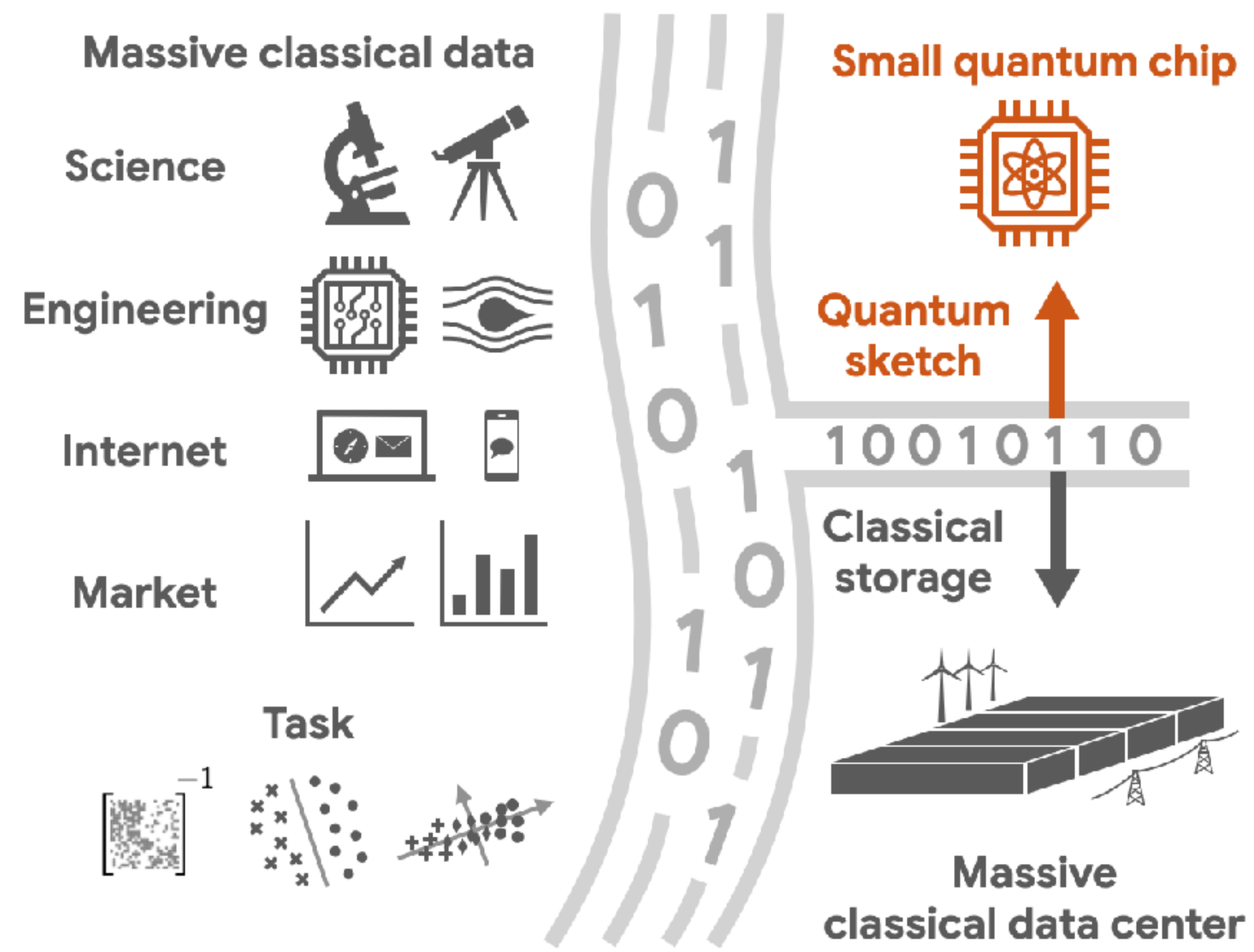
separation in query ability  
→ memory advantage

# Conclusions



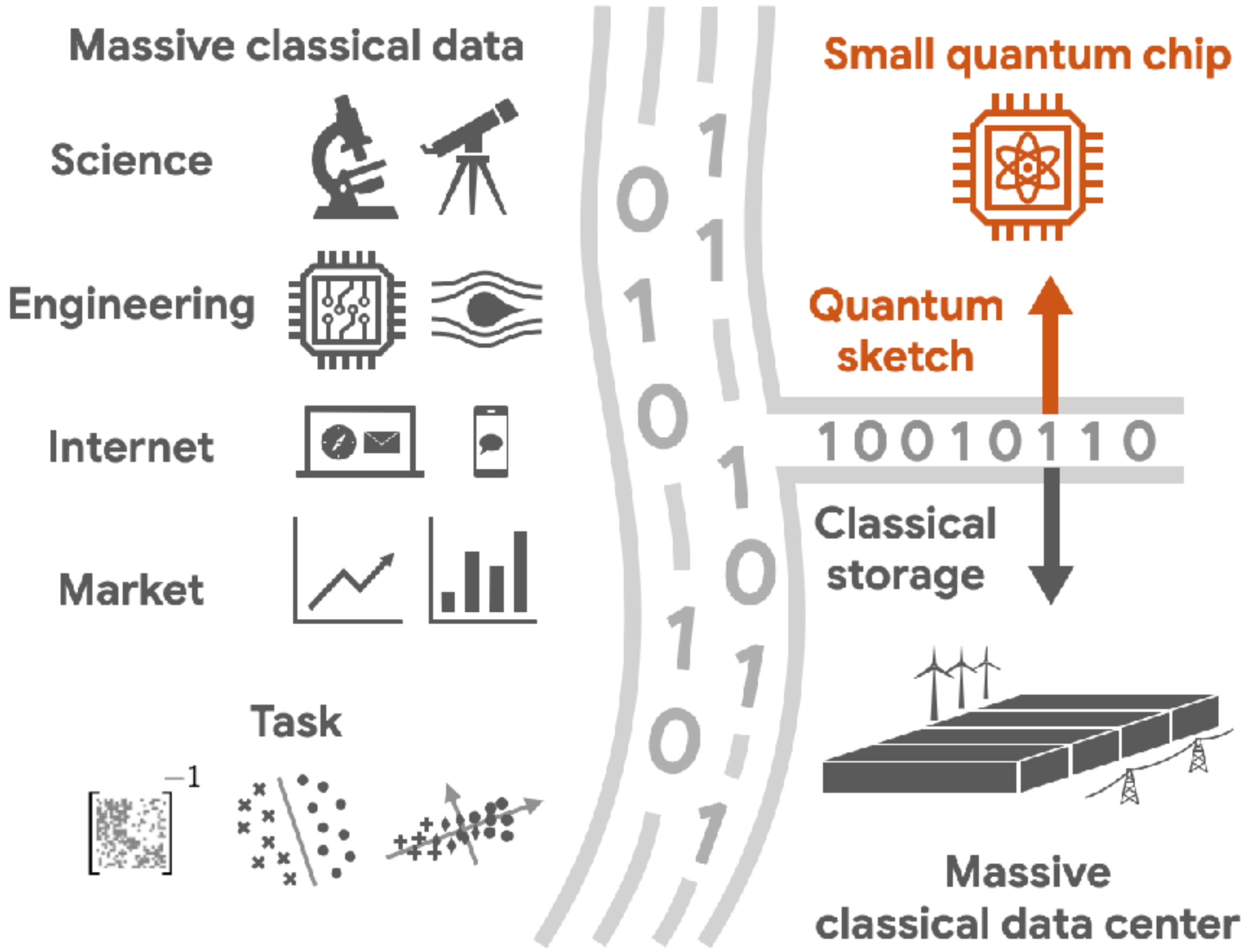
We prove that  
**small quantum machines**  
can process  
**massive classical data**  
and solve large-scale  
machine learning problems.

# Conclusions



We prove that **any classical machine** with the same performance needs **exponentially larger size** or **superpolynomially more samples.**

# Conclusions



**Our results open the possibility for quantum computers to be broadly useful in our daily life.**

# Conclusions

This exponential quantum advantage relies solely on the **correctness of quantum mechanics**, persisting even if  $P=NP$ ,  $BPP=BQP$ , etc.

Given  
sufficient  
time



**small  
quantum chip**

massive classical  
data center

# Conclusions

This **exponential** quantum advantage relies solely on the **correctness of quantum mechanics**, persisting even if  $P=NP$ ,  $BPP=BQP$ , etc.

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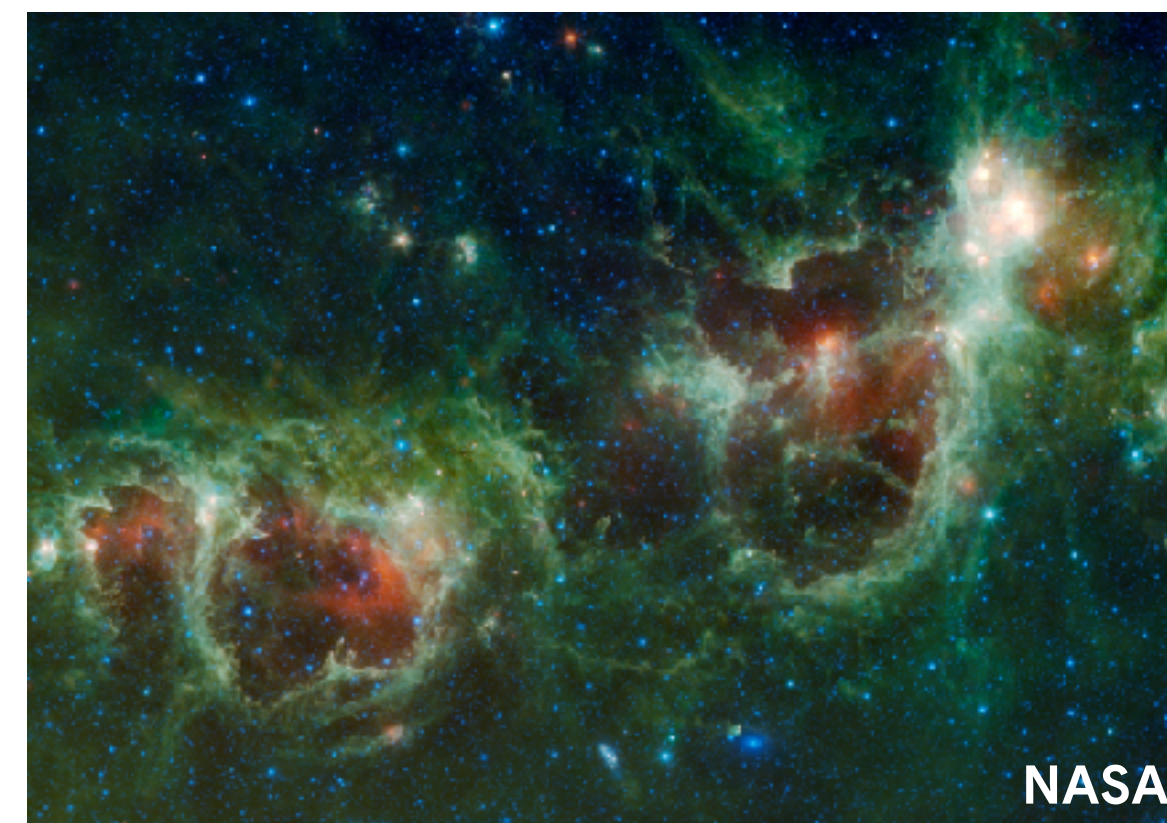
**hundreds of  
logical qubits**

universe-scale  
classical computer

# Conclusions

Machine learning offers a **fundamental test** of quantum mechanics at the **complexity frontier**.

Given  
sufficient  
time



**hundreds of  
logical qubits**

universe-scale  
classical computer